Zero Leverage and The Value in Waiting to Issue Debt

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ABSTRACT

This article documents that the real option to have debt motivates some firms to remain debt-free, even when standard trade-off theory predicts that these firms should have leverage. The real option has a first-order effect similar to bankruptcy costs in addressing the zero-leverage puzzle, the observation that many firms seemingly forgo sizable debt benefits by remaining debt-free. The debt-free firms' value includes the option whose value is derived from future debt benefits and hedging bankruptcy costs. This article proposes an optimal timing model for having debt and finds support for the model's predictions through simulations and empirical analysis.

Keywords: Zero leverage, Real option, Optimal capital structure

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1 Introduction

Almost one in five public US firms do not have debt. These zero-leverage firms seemingly ignore significant tax savings associated with the debt financing that is about 7% to 15% of their unlevered value (Strebulaev and Yang, 2013). It is a large inefficiency based on the trade-off theory because firms gain a positive value in cash flows by trading off costs and savings from having debt. For example, a firm financed with debt benefits from interest tax savings, while it faces financial distress costs. I propose that zero-leverage behavior is optimal due to the value in waiting to have debt and hedging debt costs. Hence, this article contributes to the literature on the zero-leverage puzzle. I present a simple model for timing a firm's recapitalization with optimal leverage and default. Within the reasonable parameter ranges, the model simulations generate zero-leverage observations close to the empirical rates which is about 20% while the traditional models recommend having leverage. Therefore, the models that ignore the timing real-option tend to overestimate the leverage. From model calibrations, I find the factors that lead to zero leverage: high asset volatility,[1] high debt costs, low tax levels, low payout rate, and small size. Empirically, I verify the factors by estimating both survival and choice regressions on zero-leverage (ZL) firms while I control for other variables such as cash holdings and governance. In an out-of-sample test, I also check the factors' success at predicting the choice of the firms that always have debt.

The trade-off theory identifies debt as a preferred source of financing and the earlier related theoretical studies mostly focus on explaining the observed average leverage. But, they have difficulty to match the leverage cross-section, especially the choice of zero leverage (Strebulaev and Yang, 2013). For example, Strebulaev (2007) simulates cross-sectional optimal leverage of the firms based on a dynamic trade-off model similar to Goldstein, Ju, and Leland (2001) and Leland (1998) and no firm in the simulation follows zero leverage. However, Strebulaev and Yang (2013) and Bessler, Drobetz, Haller, and Meier (2013) document a significant number, and an increasing trend, of firms with zero leverage. Every year

[1] Asset volatility is the standard deviation of the return on the unlevered assets due to business risk.

between 1962 and 2009, 10% of public US firms on average had no debt. The rate has been above 19% since 2004. This is particularly striking given that Korteweg (2010) reports a net debt benefit (total savings minus total costs) averaging 5% of a firm's value. The potential gain from having debt at the optimal leverage is 2% of the firm's value, on average, even for ZL firms that supposedly face more issuance costs in the form of financial constraints.[2] Therefore, It seems that ZL firms "leave considerable amount of money on the table by not levering up" (Strebulaev and Yang, 2013).

At first glance, the evidence is inconsistent with standard trade-off models. It also does not fit in the other frameworks, such as financial flexibility and private costs of debt. For example, a large number of ZL firms pay dividends, which is inconsistent with remaining financially flexible. In a parsimonious approach, this article abstracts from the other frameworks and only focuses on trade-off theory to check how well it can provide testable explanations about ZL policy. I model a ZL firm that holds a valuable real option for having debt. Once the firm decides to exercise the option, it chooses the optimal leverage according to some classical trade-off models. Taking on debt (the exercise of the option) has positive net benefits for the firm at any point in time. Therefore, the classical models expect the firm to be levered. However, in this article's model the firm optimally remains debt-free due to the value of the real option. The option's value reflects the hesitation that managers feel to restructure the capital because of the uncertain future of the firm and debt costs.[3] Managers only exercise the real option when doing so makes up for the loss of optionality, or hedging value of not having debt; if the firm value does suddenly decline, not having debt hedges exposure to non-recoverable and irreversible debt costs such as default. When a firm experiences downfall, it is also less likely to afford debt buyback or call. In a nutshell, the mechanism can be understood from the following example: consider an all-equity firm valued at \$100 in a Bernoulli trial. Suppose the firm's net gain is \$2 from present value

[2]They are non-dividend-paying ZL firms that are more likely to be financially constrained (Korteweg, 2010).

[3]This intuition is in line with Graham and Harvey (2001)'s survey where they find almost half of the CFOs care about the uncertainty and more than one-third consider debt costs in their debt policies.

of tax savings minus default costs, if the firm optimally recapitalizes equity with debt now. But, if the optimal leverage in the future yields net gains of \$10 with 50% chance and \$0 with 50% chance, the firm will hold the real option valued at \$5 and have zero leverage. The market also incorporates the option in its assessment of the firm's value. Thus, the real option causes the firm to deviate from traditional optimal leverage and remain debt-free.

The real-option view identifies at least five factors that make firms more likely to have zero leverage under conditions which the traditional view expects them to have leverage: high volatility, small size, low payout rate, low tax rate, and high debt costs (bankruptcy and issuance costs that do not exceed the tax savings). Most importantly, high asset volatility increases the hedging value of the option to have debt later. Let's consider two firms, one with riskier assets than the other, *ceteris paribus*. While the low risk firm issues debt, the riskier firm prefers to hold the real option. Hence, above a volatility boundary, the firm waits to have debt because its underlying asset's volatility increases the option's value. In the volatility-leverage relation, the classical models already predict low leverage for the firms with high asset volatility due to their chance of default. This article shows a jump to zero leverage due to the real option friction which creates a kink in the relation. Therefore, the classical models without the real option tend to overestimate the optimal leverage. Moreover, high debt costs such as default costs increase the propensity to have zero leverage when I limit the model with the real option to the empirical ranges around an average of 40%. However, without a real option, the classical models yield ZL policy only when the bankruptcy-cost average increases from 40% to 90% at default, which is not a realistic cost. For example, according to Glover (2016), average empirical bankruptcy cost is about 45% and less than 5% of the firms may face default costs above 90%. Hence, the real option has a first-order effect as strong as the bankruptcy costs on the choice of ZL policy and the implications of the real-option model are consistent with the earlier evidence.

This article also tests the model's implications empirically. The sample of ZL firms has the same selection criteria as the earlier empirical studies related to the ZL puzzle; these studies show that there is a net positive gain in having debt for the firms and, hence, their ZL policy is puzzling. In the 1996-2015 period, a preliminary comparison between ZL and levered firms shows that volatility is higher for ZL firms, both in overall and in size-ranked subsamples consistent with the theoretical model. Then, the article estimates two regression models: a survival regression for the duration that firms stay debt-free and a binary-choice regression to opt in or out of ZL policy. Both regressions verify that the five factors increase the duration and likelihood of remaining debt-free. The regressions control for the other factors such as financial flexibility and internal funding. In the choice regression, re-interpreting the latent choice factor yields an average estimate for the volatility boundary that separates ZL and non-ZL states of the firms and creates the kink in the volatility-leverage relation. In the same period, an out-of-sample test of the factors with the choice regression performs reasonably in predicting the choice of the firms that always have debt. The factors also pass several robustness checks including an alternative proxy for ZL firms, the subsample of firms with positive payout, and controlling for governance.

In more details, this article's model is based on original trade-off models with endogenous default and capital structure, where it adds the real option and small fixed costs to produce non-convexity. Only facing non-convex costs which do not exceed the tax savings is sufficient to create ZL policy. The debt-free firm considers optimally replacing some equity with debt and solely trades off tax savings with dead-weight bankruptcy and debt issuance costs. Traditional models focus on the optimal leverage, conditional on immediate issuance. Instead, this article considers the schedule of optimal leverage to be conditional on an optimal issuance time, which creates a waiting real option. The real-option idea extends to a wide range of classical trade-off models, dynamic or static. I analyze and simulate at least one case that results in ZL policy. The case is based on Leland (1998), which represents dynamic capital structure with issuance costs, debt rollover, and rebalancing. Having very small leverage, or debt, with very frequent rebalancing is not optimal due to transaction costs. The model calibrations yield five hypotheses for testing the model's implications empirically. I also simulate the leverage of the firms with parameters that match the earlier empirical studies in a process similar to Strebulaev (2007).[4] The model simulations show on average 23% of the firms in each quarter follow zero leverage which is close to the empirical observations. The comparable traditional dynamic model without the real option does not produce any ZL observations. Thus, when standard trade-off theory predicts that the firms should have leverage, debt-free firms can exist under empirically observed conditions.

This article makes two contributions to the trade-off literature. First, ZL firms compose an important subsample of all the firms in the leverage analysis, yet most of the traditional trade-off models are not successful in explaining ZL policy. This article provides a means of reconciling ZL policy with trade-off theory that is embeddable in the earlier models, dynamic or static, by suggesting a new mechanism borrowed from the real option theory. For example, in the simulations, the model with the real option creates results that nest the traditional model. Therefore, the article extends the inaction area from real-option literature, e.g Bloom (2009), to the trade-off and capital structure theory. Second, the existing studies on ZL phenomenon propose additional costs or restrictions on having debt that make the overall costs greater than the tax savings.[5] However, the empirical works still find positive net benefits from having debt for many ZL firms with these constraints and costs and highlight the puzzle.[6] To reconcile both streams, this article shows how the real-option feature of having debt explains ZL without assuming that these additional costs exceed the tax savings, which seems to be required by the trade-off theory. As long as these costs are concave and irreversible, they contribute to ZL behavior. Thus, under weaker conditions, debt-free firms can exist.

This article also complements the literature related to financial flexibility. DeAngelo, DeAngelo, and Whited (2011) show that financially-constrained firms retain part of their debt capacity to ensure the financing of future projects. Kisser (2013) suggests a similar

[4]Compared to Strebulaev (2007), I match the parameters such as volatility to all the firms rather than focusing only on the levered rated firms.

[5]See for example Sundaresan, Wang, and Yang (2015), Sufi (2009), and Luciano and Nicodano (2014).

[6]See Strebulaev and Yang (2013), Korteweg (2010), and Bessler et al. (2013).

motivation for cash holdings. Despite these considerations, Strebulaev and Yang (2013) find that one-third of ZL firms pay out some of their cash, either through share repurchases or cash dividends.[7] The ratio is 43% in the 1996-2015 period. These firms not only pay higher taxes by replacing interest payments with dividends, but also appear less likely to be financially constrained according to Dai, Shackelford, Zhang, and Chen (2013). For this literature, explaining ZL firms, especially with dividend payments, is "perhaps the most significant challenge"(Denis, 2012). The model here helps reconcile the existence of such firms with the flexibility arguments. Although payouts are exogenous in the model, this article shows that a dividend-paying firm may very well remain optimally debt-free to hedge default costs; bankruptcy hedging complements financial flexibility in explaining zero leverage.

The contributions of this article to the empirical studies on ZL firms are three. First, this article analyzes the determinants of the duration that ZL firms stay unlevered. Second, the choice regression re-interprets the pure statistical choice factor intuitively to derive the volatility boundary, or the kink, between ZL and non-ZL states. The median of the volatility boundary is 30% for ZL firm-quarters while their median volatility is 55%, well above the boundary. The result is in line with the model's intuition about the ZL firm-quarters which stay above their boundary. Third, the out-of-sample test cross-sectionally validates the five factors in predicting the choice of the firms that have debt all the time.

The article is organized as follows: section two models the optimal strategy to have debt. Section three empirically tests the model predictions. Finally, section four concludes.

2 Model and Hypotheses

Consider a firm with assets that produce operating income (EBIT) following Geometric Brownian Motion (GBM) under physical and risk-adjusted (RA or risk-neutral) measures. The asset value is the expected present value of all the future cashflows from the asset

[7] For earlier works, see Gorbenko and Strebulaev (2010), Gamba and Triantis (2008), Hennessy and Whited (2005),and Trigeorgis (1993).

operations and it does not depend on the financing of the asset. Therefore, the unlevered asset-value process, ν, also follows GBM under the RA measure[8]:

$$\frac{d\nu}{\nu} = (\mu - \delta)dt + \sigma dW^p \quad \Leftrightarrow \quad \frac{d\nu}{\nu} = (r - \delta)dt + \sigma dW^q \tag{1}$$

where δ is the asset payout rate, σ is the standard deviation or volatility, r is the risk-free rate, μ is the historical drift and $\mu - r$ is the asset risk premium. Both processes W^p (physical shocks) and W^q (RA shocks) are standard Brownian motions in their respective measures. They are related to each other with Girsanov's theorem. Initial unlevered asset value is ν_0. The firm is an all-equity firm. The firm's managers decide when to optimally issue debt to recapitalize. The optimal-timing problem's state variable is the unlevered value which is lean, without the real-option value.

When the firm recapitalizes, debt optimally replaces equity . I leave more details about debt, e.g. debt structure (dynamic or static) and net benefits function, to the cases analyzed later. Equity holders cash the equivalent of the replaced equity with the inflow of debt. The recapitalization also creates an extra value as the net debt benefits which is equal to tax savings minus debt costs. According to trade-off theory and adjusted present-value (APV), the extra value adds directly to the shareholders' wealth. Hence, having debt in the model is solely to save on taxes and has no other objectives such as financing new projects.

Another way of looking at the recapitalization is from the eyes of the firm's managers who aim to increase the firm's total value. In a Modigliani and Miller (1958) framework, tax savings from interest payments, debt issuance and default costs are the only frictions. The financing method does not affect the operating income and the model assumes for simplicity that there is no agency problem or information asymmetry. Hence, the managers care about the extra gain from net debt benefits. Prior to having debt, the managers manage an unlevered firm. But, after having debt, they manage the same firm with an extra value added by net debt benefits. Thus, the recapitalization is similar to a positive NPV project.

[8]See Appendix A for the proof. Appendix B has the list of the variables used in all the models.

The tax savings are similar to the present value of the future incomes. The debt costs are similar to the present value of the possible future costs. The net debt benefits are equal to NPV which is also the exercise value of the option. Not every positive NPV project with a waiting real option is started by the managers. For the managers, the question is when, or in which state, to have debt and implement this positive NPV project. The real-option value quantifies the hesitation that managers feel to restructure the capital because of the uncertain future of the firm and riskiness of its assets. They have to find the state with the largest expected NPV from issuing debt, the time when the net gain is higher than the option's hedging value.

The model recruits some of the assumptions from the classical models[9]: a) Debt has a long maturity. Scherr and Hulburt (2001) report an average debt maturity between 20 to 30 years for levered small firms. A short-term debt has high costs such as rollover risk and covenants. Since it adds an extra dimension to the model's complexity, this article assumes constant maturity and does not consider optimal maturity. b) Debt is irreversible due to no downward restructuring and its costs such as issuance costs are not recoverable. The irreversibility assumption naturally holds as long as the costs to buyback debt are higher than default or issuing new debt. Default is not uncommon and classical hold-out problem makes refinancing very costly. There is also empirical evidence about the irreversibility assumption. Korteweg (2010) empirically finds that the firms which gain the most from debt reduction are the least likely to buyback. Gilson (1997) mentions transaction cost as one of the reasons behind the irreversibility of debt. The next two assumptions are for simplicity: c) All the parameters are exogenous and constant including the payout rate and the volatility. The volatility reflects the business risk of the firm which is not controlled by the financial manager and exogenously depends on the business-environment risk. The asset payout rate is the leak in the asset value, such as sticky payout to shareholders or convenience yield of the products sold. The payout is positive, exogenous and constant. d)

[9]The assumptions are shared with many other capital structure models, such as Leland and Toft (1996), Bhamra, Kuehn, and Strebulaev (2010), Broadie, Chernov, and Sundaresan (2007), and Glover (2016).

The asset captures the random shocks from one source of risk which also affects the EBIT.

The model also borrows some assumptions from the real-option literature (for example, see Dixit and Pindyck (2012)). e) The real option has no expiration. Hence, the time homogeneity makes it possible to derive closed-form solutions for the option and optimal exercise policy. Finite option maturity does not change the theoretical results and implied hypotheses but demands numerical methods for valuation. f) A condition satisfied throughout this article's analyses, such as the calibrations and simulations, is that debt issuance is feasible; it has positive NPV in all the analyzed states which means tax benefits are always larger than the debt costs.[10] Otherwise, if having debt would destroy value with negative NPV, there would be no ZL puzzle.

[Place Figure 1 about here]

Since debt issuance has real option properties by the assumptions, I extend the real-option analysis as discussed in McDonald and Siegel (1986) and Dixit and Pindyck (2012) on having debt. For a positive NPV project with a waiting real option, it is not optimal to immediately start the project merely because NPV is positive. It is optimal to exercise the waiting option and start the project only when the project's income reaches above a certain threshold which maximizes the expected NPV and hedges against the project costs. Similarly, there is an unlevered asset value, ν_I, above which recapitalization and debt issuance become optimal. Figure 1 explains the model structure. For the time being, the net debt benefit is a general function on the unlevered value of the firm, $DB(\nu)$. The optimal condition for exercising the real option is the only subject of interest: "what is the optimal threshold to recapitalize and issue debt?" I use the contingent-claim approach of Ericsson and Reneby (1998) to value the option. The option's value in the continuous state space depends on the value of the contingent claim, F, paying \$1 at the recapitalization threshold. The claim satisfies the following Black-Scholes partial differential equation (PDE) with the Dirichlet boundary

[10] The real option is in-the-money. Even if debt cost are higher than tax savings, the out-of-the-money option has value. But, I abstract from analyzing it for brevity.

condition at the threshold:

$$\frac{1}{2}\sigma^2\nu^2 F_{\nu\nu} + (r-\delta)\nu F_\nu - rF = 0, \qquad F(\nu_I) = 1, \qquad F(\nu) = (\frac{\nu}{\nu_I})^{\beta_1} \tag{2}$$

where F_ν is the partial derivative with respect to ν. The perpetual real option's value is the contingent claim's value times the net debt benefit at the threshold:

$$\begin{aligned} &W(\nu) = DB(\nu_I)(\frac{\nu}{\nu_I})^{\beta_1} \qquad \nu \leq \nu_I, \\ &DB(\nu) = TS(\nu) - DC(\nu), \qquad \beta_1 = \frac{\sqrt{h^2+2r}-h}{\sigma}, \qquad h = \frac{r-\delta-(\frac{1}{2}\sigma^2)}{\sigma} \end{aligned} \tag{3}$$

where DB is the present value of net debt benefit, TS is the present value of tax savings, DC is the present value of debt costs, and W is the present value of the option to wait. The option's formula has two elements: the last part is the value of the contingent claim, and the first part is the net debt benefit at issuance. In the Dixit and Pindyck (2012)'s sense, the first part of the net debt benefit, DB, is the project revenues, TS; and the last part is the project costs, DC. Analogous to an American call option, TS is similar to the value of stock, DC is similar to the exercise price, and $TS - DC$ is the option's exercise payoff. Table 1 presents further comparison between the options. The real option calculates the discounted expected value of the benefits at the optimal exercise threshold. The total market value of the firm before having debt is the unlevered lean value plus the option, $\nu + W$. The optimal debt issuance threshold is set to maximize the total value of the firm, $\nu_I : \underset{\nu_I}{Max}\ \nu + W$. Solving the implied first-order condition (FOC) results in:

$$\nu_I : \quad \frac{\partial DB(\nu_I)}{\partial \nu_I} = \beta_1 \frac{DB(\nu_I)}{\nu_I} \quad \Leftrightarrow \quad (\frac{\partial DB(\nu_I)}{\partial \nu_I} / \frac{DB(\nu_I)}{\nu_I}) = \beta_1 \tag{4}$$

[Place Table 1 about here]

The optimal threshold satisfies both equations. Analytically, the left-hand side of the first equation is the marginal benefits of waiting for the unlevered value to increase. The right-

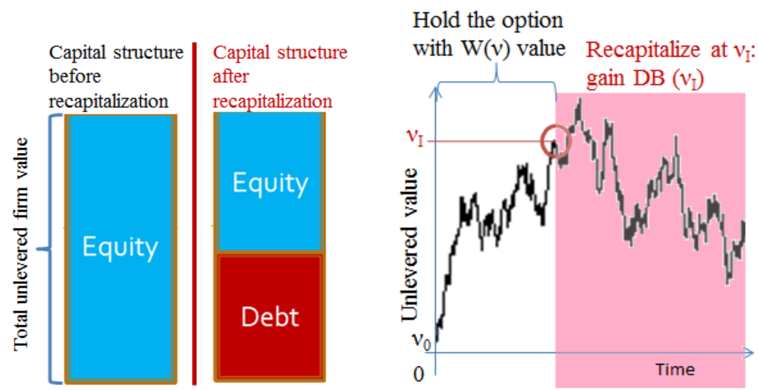


Figure 1. The model setup

Table 1- Analogy between the option to have debt, an American perpetual call option, and perpetual waiting option to start a project: X is the exercise price for the call. S is the stock price. V is the present value of the project income. B is the project's constant setup cost. S and V as the underlying securities are analogous to ν. FOC is the first-order condition in the problem to find the exercise threshold that maximizes the option value.

Option	Exercise value	Option value	FOC for optimal threshold
Having debt	$DB(\nu) = TS(\nu) - DC(\nu)$	$DB(\nu_I)(\frac{\nu}{\nu_I})^{\beta_1}$	$\nu_I^* = \beta_1 \frac{DB(\nu_I^*)}{\frac{\partial DB(\nu_I^*)}{\partial \nu_I^*}}$
American perpetual call	$C(S) = S - X$	$C(S_I)(\frac{S}{S_I})^{\beta_1}$	$S_I^* = \beta_1 \frac{S_I^* - X}{1}$
Perpetual waiting option	$NPV(V) = V - B$	$NPV(V_I)(\frac{V}{V_I})^{\beta_1}$	$V_I^* = \beta_1 \frac{V_I^* - B}{1}$

hand side of the first equation is the average benefits at the current level of the unlevered value. For the unlevered values below the threshold, the marginal benefit on waiting is higher than the average benefit from having debt. Thus, managers prefer to wait for the unlevered value to increase. The threshold is the point at which the marginal and average benefits match. Figure 2 illustrates the concept. Beyond the threshold, not exercising the option creates loss for the firm because the average benefit surpasses the marginal benefit and waiting has no gain. In Equation 4, the second equation implies that the marginal benefit divided by the average benefit has to match β_1. The ratio is the elasticity of the net debt benefits with respect to the unlevered value. Below the threshold, the elasticity is larger than one and waiting is optimal.

[**Place Figure 2 about here**]

The ν_I/ν_0 ratio is the waiting ratio: a ratio below 1 implies immediate exercise and large waiting ratio above 1 implies long waiting time for the process to hit the threshold. Prior to crossing the threshold, the firm benefits from simply holding the real option. This inactive option has value for the firm and no value is left on the table. Thus, the option answers the concern about firms' ignoring net debt benefits. Indeed, the ZL firms do not ignore the value in the net debt benefits; they only prefer to keep the option alive.

In the next section, the article applies the model to a realistic case with dynamic capital structure. In Appendix G, I also show that the model applies to static trade-off models and model implications are similar.

2.1 Case I: The real option to have debt in dynamic capital structure

The earlier model considers a general form for the debt structure that can be dynamic or static to convey the real option idea. Case I shows that the real option idea applies to the traditional dynamic trade-off models. The dynamic capital structure model in Case I includes features such as dynamic leverage, debt rollover, debt retirement, limited maturity and rebalancing. Consider an all-equity firm with the option to recapitalize with debt according to the following dynamic capital structure model similar to Leland (1998): Debt is issued for the first time in a lump sum to recapitalize equity. After issuance, there is continuous debt rollover. The firm partially retires debt at a continuous rate m (the average debt maturity is $M = 1/m$) and issues new debt with identical coupon c and face value p to replace it. In sum, the firm has outstanding debt with coupon payments C and face value P at any point in time. Since overall debt, P, is retired at rate, m, the newly issued debt has to be $p = mP$. Due to the rollover, the total debt service is the overall coupon rate, C, plus the net partial principal repayment, mP. For example, if there is no debt rollover with rate 0, the debt turns into a consol bond. The debt rollover creates a continuous but small transaction cost at rate k_2 times the retired principal of debt, mP.

The outstanding debt structure remains the same until the firm either decides to rebalance debt upwards at a higher asset value, ν_U, or file for bankruptcy, at ν_B. Bankruptcy costs are proportional to the unlevered assets' value at default, $\alpha\nu_B$.[11] Issuing debt for the first time and rebalancing is also costly. At the time of the first issuance, the cost is the variable cost plus the fixed cost, $k_1 P + \kappa$. It is common in the literature to have issuance costs such as in Fischer, Heinkel, and Zechner (1989), Hackbarth, Hennessy, and Leland (2007), Gamba and Triantis (2008) and Hackbarth and Mauer (2012). These costs are larger than the rollover cost and keep growing at the same rate as the value of the firm in the rebalancing points, $\phi = \nu_U/\nu_0$. At the rebalancing point, the new debt is issued at a new face-value that is proportional to the earlier principal, ϕP and its cost is $\phi(\kappa + k_1 P)$. The rebalancing keeps happening whenever the unlevered value hits the upper limit which depends on ϕ. Technically, higher costs imply less discrete rebalancing as indeed happens in reality. Having very small leverage is also not optimal because it creates higher rebalancing frequency and

[11] The assumption is about the costs that the firm will face when it decides not to serve debt and file for bankruptcy. One can also assume these costs are the costs that the firm will face when it decides to start negotiating with debt holders for debt relief, such as hold-out problem, lost sales, lost customers, or reputation cost. When creditors acquire the levered firm at default, the firm becomes an all-equity firm and has the real option to issue the debt again. However, this gain is very small compared to the default costs that the new owners pay and dropping the gain does not affect the results.

issuance costs. It is analogous to portfolio rebalancing problem with issuance cost where frequent rebalancing is not optimal. The small fixed issuance cost creates concave debt costs and a convex debt-benefit function. The debt structure remains homogeneous over time even at the issuance for tractability. As a result of this assumption, the model is simplified.[12]

The managers optimally make four decisions: a) debt issuance threshold, ν_I, which determines when to have debt (only in the model with the real option to have debt) b) the amount of debt for leverage, P,[13] c) the default barrier, ν_B, which determines when to default d) the debt rebalancing ratio, ϕ. The managers trade-off between tax savings and bankruptcy costs to find the optimal leverage and the optimal rebalancing point. The optimal default barrier is activated after having leverage. The decision variable for the default barrier is the unlevered asset value below which the shareholders will stop serving the coupons.

Figure 3 shows the setup for the model. Appendix C has more details about the model and the derivation of the Case-II formulas. Present value of debt benefit, DB, tax savings, TS, and debt costs, DC are:

$$DC(\nu) = BC(\nu) + RC(\nu), \quad DB(\nu) = TS(\nu) - DC(\nu),$$
$$\begin{cases} TS(\nu) = \dfrac{\tau C}{r} + A_T\nu^{\beta_1} + B_T\nu^{-\beta_2}, \qquad BC(\nu) = A_B\nu^{\beta_1} + B_B\nu^{-\beta_2} \\ RC(\nu) = k_1P + \dfrac{k_2mP}{r} + \kappa + A_R\nu^{\beta_1} + B_R\nu^{-\beta_2} \end{cases} \tag{5}$$

where RC is the present value of rebalancing and roll over costs, BC is the present value of bankruptcy cost, and τ is the tax rate. All the other parameters such as β_2, A_T, B_T are defined in Appendix C. Managers choose the default barrier to maximize equity value. They choose the optimal rebalancing point and the optimal leverage to maximize the total debt benefits for the firm in line with Goldstein et al. (2001). The optimal default barrier,

[12] In order to start this debt structure at the issuance, the firm issues a lump-sum debt with the same face value, P, and average maturity, $1/m$. Every infinitesimal part of the total debt has a different maturity similar to the future debt structure.

[13] Similar to Leland (1998), Case I assumes the coupon rate is the risk-free rate times the principal. If both coupon and face value were optimized, the managers issue a consol bond to avoid the rollover risk and the model collapses into the static model presented in Appendix G.

leverage, and rebalancing points has no closed form solution and are calculated numerically. The model is a classic dynamic trade-off model up to this point; managers only make decisions about three variables in the traditional models. However, the model with the real option has the optimal threshold, the fourth decision variable. I numerically calculate the optimal threshold to have debt, ν_I, by solving Equation 4 where $DB(\nu) = TS(\nu) - DC(\nu)$ from Equation 5. Equation 4 will hold for many other traditional models, if the contingent claim paying $1 at the issuance has a power-function form. The optimal default, rebalancing and leverage depend on when the firm decides to have debt. While the decisions chronologically happen as having leverage, choosing the optimal leverage, rebalancing and default, the modeling strategy is actually to move backwards. After finding the optimal default barrier, the model calculates the optimal rebalancing and leverage and, finally, the optimal issuance threshold. The threshold depends on the firm characteristics, such as the unlevered value, asset volatility, $\nu_I(\nu_0, \sigma, \alpha, \tau, ...)$, and it is ready for simulation and calibration.

[Place Figure 3 about here]

The model in Case I without the real option to have debt is very close to the models in Leland (1998) and Goldstein et al. (2001). Compared to Leland (1998), the model shares most of the features such as optimal leverage and rebalancing in the future. But, I drop optimal risk shifting and cash-flow-triggered default for the sake of simplicity and having a parsimonious model. For example, if there is the possibility of going bankrupt for a profitable business due to cash-flow triggers, the managers are more likely to hold the real option to hedge default costs because the costs of debt go higher. The model in Case I includes the option to wait and non-convex debt costs in form of a small fixed issuance cost, κ, at the time of rebalancing and issuance. As I show later in the simulation for the traditional model, adding the non-convex cost assumption to the original Leland (1998)'s model still results in issuing debt right away and does not imply zero leverage, if the net debt benefits are positive. Zero leverage needs to be handled separately using the optimal timing formula presented in this article's Equation 4.

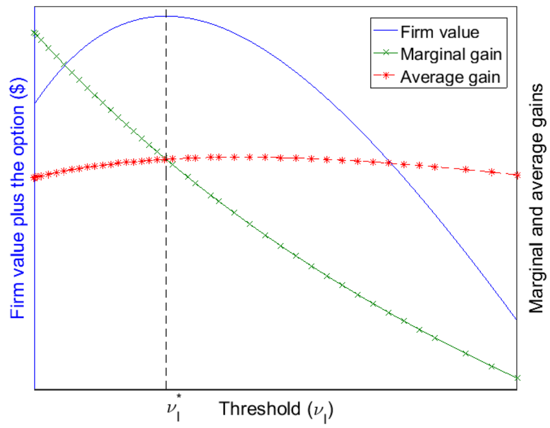


Figure 2. The optimal threshold to have debt and its derivation
X-axis shows the threshold values. Left Y-axis shows the total value of the firm. Right Y-axis shows the marginal and average net debt benefits. The total value is the unlevered value, ν, plus the option, $W(\nu)$. The optimal threshold is the point that maximizes the total value, or where marginal and average debt benefits meet. When the unlevered value increases to this point, the wait is over and the firm issues debt.

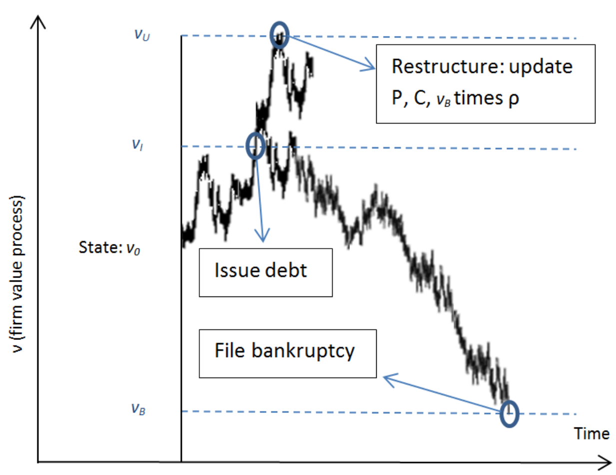


Figure 3. The model setup in Case II

2.1.1 Simulating the dynamic model with the real option in Case I

One of the main results of the model simulation is about generating ZL policy when compared to the traditional model. In the firm-quarter data simulation, the dynamic model with the real option to have debt creates ZL statistics closer to the empirical observations. Following the steps similar to Strebulaev (2007), I simulate optimal leverage paths of 500 firms in 1000 economies for 201 quarters while the underlying model is Case I. In each path for each firm, the underlying unlevered value follows the GBM. The firm updates its capital structure with the optimal decisions, such as optimal threshold to have debt and optimal leverage, which are calculated and known at time 0.[14] There are two simulations setups: The traditional model without the real option and the same model with the real option to have debt. At the initial quarter in the model with the real option (time 0), the firm chooses the optimal timing of having debt according to the optimal threshold. If the unlevered value is below the threshold, the firm follows ZL policy until the threshold is hit, but it recapitalizes with debt according to the optimal leverage if the value is above the threshold. At the issuance, the capital structure is similar to the traditional model. Once debt is issued, the firm remains levered and rebalances the debt at each optimal rebalancing point and all the costs are updated accordingly. In case of default, equity holders lose the control of the firm to the creditors after the bankruptcy costs are deducted. The new firm follows the optimal strategy for having debt.

In the simulation for the model without the real option, the firm ignores the optimal threshold to wait and recapitalizes immediately at optimal leverage since the benefits are positive. But, rebalancing and default follow the optimal choice similar to the model with the real option. When the firm hits default, the new firm is controlled by the creditors who take on optimal leverage. If at this point debt issuance does not have positive net benefits

[14] In the simulation, the GBM shock in Equation 1 is a linear combination of two independent shocks: $\sigma dW^q = \sigma_O dW^O + \beta\sigma_S dW^S$ where dW^O is the idiosyncratic firm shock and dW^S is the systematic economy-wide shock. The firm is related to economy-wide shocks with the asset beta, β. Total asset volatility, σ, is $(\sigma_O^2 + (\beta\sigma_S)^2)^{0.5}$, where σ_S and σ_O are standard deviations of the systematic and idiosyncratic shocks respectively.

due to very small size of the firm relative to the fixed issuance cost, the firm is replaced by the original firm at the first quarter; it is about 0.2% of the simulations. Therefore, debt issuance has positive NPV at the beginning and all of the restructuring points for both models. More details are available in Appendix D.

Parameter values for the simulation are presented in Table 2. Both models with and without the real option to have debt share the same parameters and underlying random processes. The parameter values for systematic volatility, asset beta, asset risk premiums, risk-free rate, asset payout rate, and the shape of the distributions for the parameters, such as asset volatility, and the debt rollover, issuance and bankruptcy costs, are similar to Strebulaev (2007). For example, all the costs follow acute Trapezoidal distribution.[15] The mean and standard deviation of the asset volatility are close to the empirical values calculated for all the firm-quarters between 1996 and 2015 in the empirical sample (see Section 3.1.1). The average asset volatility in Strebulaev (2007) is 0.25 while the average in Table 2 is 0.39. The difference is because his study and similar other studies limit the volatility distribution to only the rated firms with leverage which are reported in Schaefer and Strebulaev (2005). But, this article has the sample of all available firms and the average volatility is higher when the sample includes non-rated firms and ZL firms. The average volatility is close to the studies with a similar sample such as Elkamhi, Ericsson, and Parsons (2012).[16] 95% of the empirical asset volatilities are below 0.9. Since volatility may take extreme unrealistic values, I winsorize the asset volatility at 0.9 in the simulation. Initial unlevered asset value is at 100 and it is scalable. The range for bankruptcy cost rate is between 30% to 50% with an average of 40%. It is close to 45% reported by Glover (2016).[17] The total recapitalization

[15] Where $Trap(min, max, w, b) = \mathcal{U}(min, min + (1-w)(max-min)) + w \times (max-min)b$ is the trapezoidal distribution between min and max and w is the weight to determine the shape and is set at 1/3. $\mathcal{U}(min, max)$ is the uniform distribution between min and max. b follows Uniform distribution between 0 and 1.

[16] Nevertheless, I report simulation results with average asset volatility of 27% in Online Appendix I.1. The simulation creates ZL firm-quarters which on average is about 8% of the draws.

[17] Some earlier studies use a rate between 0-20% (see e.g. Bris, Welch, and Zhu (2006)). However, Elkamhi et al. (2012) argue that it is close to 50%, if financial distress costs before filing for bankruptcy are also included, such as the cost of lost sales and customers. In addition, Glover (2016) argue that 20% cost has sample selection bias and after fixing for the bias the rate is 45% on average.

and rebalancing costs, $\kappa + k_1 P$, average about 1.17% of the debt's market value at time zero which is in the range for flotation costs reported by earlier studies, 1.09 % in Hackbarth and Mauer (2012), and 2% in Gamba and Triantis (2008). Similar to Leland (1998), the tax rate is 25% and lower than the 35% corporate tax rate for accommodating lost benefits due to personal taxes. In order to reduce the effect of the initial simulation conditions on the results, I drop the first half of the quarterly draws in the sample. The ranges for parameters result in reasonable simulation values for variables such as the net debt benefits for the firm. For example, the average net debt benefits in both of the models are about 3% with standard deviation of 1% (see the net debt benefits in Table 3). These numbers are close to empirical values and simulations reported by earlier studies such as Strebulaev (2007) and Korteweg (2010). Calibrating Case I model with the average parameters from Table 2 also result in the net debt benefits close to 3%.

[**Place Table 2 about here**]

The model with the real option does a better job in simulating leverage and ZL behavior when compared to the traditional model. The average total leverage in the model with the real option is 18% which is close to 17%, the average leverage for all firm-quarters between 1996 and 2015 in the empirical section. The real option model is capable of reproducing the reported ratios by Strebulaev and Yang (2013) which is in the range of 13% to 20% since 1996. In Table 3, about 23% of the market leverage of the firms are 0 due to the value in waiting while none of the firms are debt-free in the traditional model. The traditional model fails to generate ZL behavior in the simulation even for the extreme values such as asset volatility of 90% and bankruptcy costs of 50%. As I drop the ZL observations in the real option model, the leverage distribution looks very similar to the traditional model. Hence, the results from the real option model nests the simulation results from the traditional model. In many of the empirical studies, market leverage is calculated as the ratio of book value of debt to the sum of equity's market value and the book value of debt (Quasi-market leverage, QML). The results remain the same on the ZL behavior for QML. The results are

Table 2- Parameter values in the simulation for the dynamic model in Case I: ν_0 is the initial asset value a time 0. Asset β correlates the asset shocks with the economy-wide shocks. σ_S and σ_O are standard deviations of the systematic and idiosyncratic shocks respectively. σ is the standard deviation for the total shock to the firm's assets. δ is the asset payout rate. r is the risk-free rate. κ is the fixed issuance cost of debt. k_1 is the proportional issuance cost of debt. k_2 is the debt rollover cost. m is the debt retirement rate. α is the proportional bankruptcy costs. τ is the tax rate. $\mu - r$ is the asset risk premium. b is the parameter to create Trapezoidal distribution. $\mathcal{U}(min, max)$ is the uniform distribution between min and max values. $Trap(min, max, weight, b) = \mathcal{U}(min, min+(1-weight)(max-min)) + weight \times (max-min)b$ is the trapezoid distribution between min and max where $weight$ determines the shape. The distributions are similar to Strebulaev (2007). *: The volatility is winsorized at 0.9 since 95% of all the empirical firm-quarters have volatility below 0.9 as reported in Section 3.1.1.

Parameter	Distribution	Mean	Std
ν_0	constant	100	-
Asset β	normal	98.9%	50.2%
σ_O	$z_0 + z_1\chi^2(z_3)$	35.3%	21.2%
	$z_0 = 0.05, z_1 = 3/40, z_3 = 4$		
σ_S	constant	14.8%	-
σ	$\sqrt{\sigma_U^2 + (\beta\sigma_S)^2}$	39.1%*	18.2%
δ	$\mathcal{U}(0.02, 0.04)$	3.0%	0.58%
r	constant	5.0%	-
b	$\mathcal{U}(0, 1)$	50.8%	28.5%
κ	$Trap(0.015, 0.025, 1/3, b)$	0.02	0.2%
k_1	$Trap(0.008, 0.010, 1/3, b)$	0.9%	0.04%
k_2	$Trap(0.004, 0.006, 1/3, b)$	0.5%	0.04%
m	$Trap(0.09, 0.12, 1/3, b)$	10.5%	0.66%
α	$Trap(0.3, 0.5, 1/3, b)$	0.4021	4.3%
τ	constant	25.0%	-
$\mu - r$	constant	6.5%	-

also robust when I include all the simulated observations in the ratio calculations. Hence, the simulations of the model with the value in waiting to have debt are closer to replicating the empirical observations on the ZL behavior.

[Place Table 3 about here]

The model also shows that cash payout and ZL policy are not contradictory. In the simulations, the payout rate has an average of 3%. Payout can take many forms, such as dividend payments or share repurchases. Financial flexibility theory argues that ZL firms may hold their debt capacity to remain flexible in financing future projects. This prediction seems contradicting with the empirical literature: both Strebulaev and Yang (2013) and Korteweg (2010) show that many ZL firms pay dividends. Paying dividend implies that these firms do not have flexibility concerns. However, the model in this article shows that a firm may payout cash and follow ZL policy simultaneously to hedge default, which complements the financial flexibility theory in explaining the ZL behavior.

2.1.2 Calibrations and hypotheses

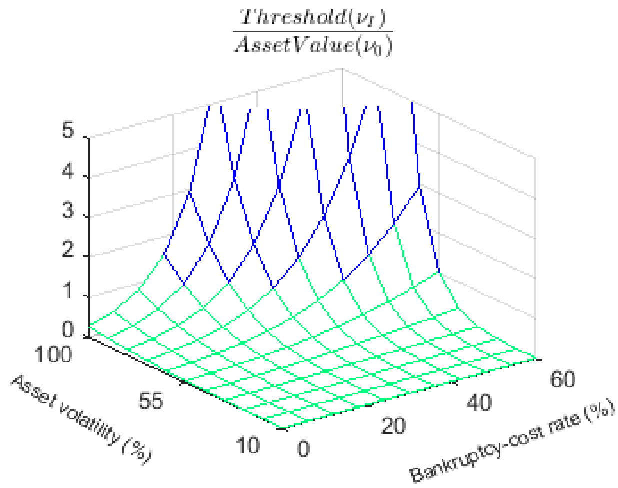
Model calibrations yield two contributions and five hypotheses for testing the model implications in the empirical section. I calibrate the model to average of the parameters reported in Table 2. In all the calibrations, having debt is feasible and net debt gain is positive. Figure 4a shows the waiting ratio in the two-dimensional (2D) plain of the volatility and bankruptcy costs. In Figure 4a, any point above 1 leads to ZL policy. Figure 4b is generated by a look to Figure 4a from the top. Technically, the contour graph for Figure 4a at 1 creates Figure 4b. Figure 4 shows the increasing trend of waiting with respect to the asset volatility and bankruptcy cost, which implies the next two hypotheses:

Hypothesis 1. *The duration and probability for a firm to follow the ZL policy are positively related to the firm's asset volatility:* $\frac{\partial(\nu_I/\nu_0)}{\partial\sigma} > 0$ *(H1).*

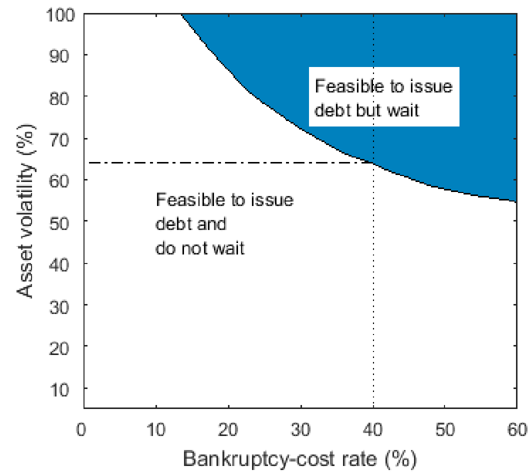
Hypothesis 2. *The duration and probability for a firm to follow the ZL policy are positively related to the firm's debt costs [but the costs are not higher than the tax savings]:* $\frac{\partial(\nu_I/\nu_0)}{\partial\alpha} > 0$

Table 3- Simulation results for the dynamic capital structure model: The table reports the distribution of the market leverage, quasi-mark leverage, zero leverage firms and average net debt benefits from the simulation. The table compares the traditional dynamic trade-off model without the real option and this article's model with the real option of optimal timing to have debt, as described in Case I. It also reports the distributions when zero leverage (ZL) observations are dropped from the model with the real option. The statistics are based on simulating observations of 1000 economies with 500 firms for 201 quarters (50 years). The first section reports the statistics after dropping the first 25 years of observations from the simulation. The mean, standard deviation and the percentiles for the leverages are reported for all the observations. Market leverage is the ratio of the market value of debt to the market value of the firm. Quasi-market leverage is the ratio of book value of debt to the sum of equity's market value and the book value of debt. Average number of ZL firms is reported after averaging the ZL observations in each quarter within each economy and then calculating the mean for all the economies. Net debt benefits are the tax savings less the debt issuance and default costs. Average benefits are reported after averaging them for each firm in each economy and then calculating the mean for all the economies. The second section includes all the observations simulated. The last section only reports the statistics of the observations across the firms in the initial quarter (time 0) for all the economies.

			Percentiles							
	Mean	Std	5	10	15	20	50	85	90	95
100 Quarters in the last 25 years of the simulated data										
Market leverage (ML)										
The real option model	0.18	0.17	0	0	0	0	0.15	0.34	0.40	0.53
The real option model without ZL	0.24	0.16	0.06	0.08	0.10	0.11	0.20	0.38	0.45	0.57
Traditional model	0.23	0.17	0.04	0.06	0.07	0.09	0.18	0.38	0.45	0.59
Quasi-Market leverage (QML)										
The real option model	0.19	0.19	0	0	0	0	0.16	0.35	0.43	0.58
The real option model without ZL	0.25	0.18	0.06	0.08	0.10	0.11	0.20	0.40	0.48	0.64
Traditional model	0.24	0.19	0.04	0.06	0.08	0.10	0.19	0.41	0.51	0.68
Average number of ZL firms in each quarter				Average net debt benefits						
	Mean	Std						Mean		Std
Traditional model	0	0		Traditional model				0.03		0.01
The real option model	0.23	0.12		The real option model				0.03		0.01
Complete simulated data including the initial values at time 0										
Market leverage (ML)										
The real option model	0.18	0.17	0	0	0	0.04	0.16	0.33	0.39	0.51
The real option model without ZL	0.22	0.16	0.05	0.07	0.09	0.10	0.19	0.36	0.42	0.54
Traditional model	0.22	0.17	0.03	0.05	0.07	0.09	0.18	0.36	0.43	0.56
Initial values at time 0										
Market leverage (ML)										
The real option model	0.15	0.10	0	0	0.06	0.07	0.14	0.25	0.27	0.32
The real option model without ZL	0.17	0.08	0.06	0.07	0.08	0.09	0.16	0.25	0.28	0.33
Traditional model	0.15	0.09	0.03	0.04	0.06	0.07	0.14	0.25	0.27	0.32



(a) The waiting ratio (ν_I/ν_0) as a function of volatility and the bankruptcy cost.



(b) ZL policy in the cross-section for volatility and the bankruptcy cost

Figure 4. ZL policy in the two-dimensional cross-section for volatility and bankruptcy cost in Case I with dynamic capital structure: The X-axis shows the proportional bankruptcy cost (PBC) rate. The Y-axis shows the firm's asset volatility. The Z-axis shows the waiting ratio as the ratio of the optimal recapitalizing threshold to the firm's value. A ratio above 1 implies that the firm waits to issue debt as seen in the contour plot. Figure 4b is generated by a look to Figure 4a from the top. In Figure 4b, dependent decision variable is to follow ZL policy. Firms in the dark area follow ZL policy even when having debt has a positive net value (feasible) and traditional model predicts that these firms should have leverage. Firms in the white area prefer to issue debt. The other parameters such as risk-free rate match the averages in Table 2.

(H2).

[Place Figure 4 about here]

[Place Figure 5 about here]

In Figure 5, waiting ratio to have debt has negative relation with tax rate, payout rate, and the size of the firm, which leads to the following hypotheses:

Hypothesis 3. *The duration and probability for a firm to follow the ZL policy are negatively related to the tax rate [but the tax savings are not lower than the debt costs]:*$\frac{\partial(\nu_I/\nu_0)}{\partial\tau} < 0$ *(H3).*

Hypothesis 4. *The duration and probability for a firm to follow the ZL policy are negatively related to the payout rate:*$\frac{\partial(\nu_I/\nu_0)}{\partial\delta} < 0$ *(H4).*

Hypothesis 5. *The duration and probability for a firm to follow the ZL policy are negatively related to the firm's size:*$\frac{\partial(\nu_I/\nu_0)}{\partial\nu_0} < 0$ *(H5).*

The intuitions in the model for the hypotheses is analogous to an American perpetual call exercise from the option's literature (see Table 4 for the analogy). Any factors that increase the hedging value of the real option contributes to a longer waiting. For example, the waiting increases with bankruptcy costs similar to the classical exercise policy of an American call where the exercise threshold increases in exercise cost. The waiting is also longer when the size of the company is small similar to an American call option with low stock prices. The result about the size also defines a proper strategy for having debt in the growth path of a firm. *Ceteris paribus,* when the firm is small and young, it is better to wait before having debt, even if it is feasible. This helps the firm to hedge the exposure to the possible bankruptcy costs. If the firm's value suddenly decline, not having debt postpones the exposure to these non-recoverable costs. Later, when the firm grows larger and matures enough to reduce the default chance and costs, it is time to have debt.

[Place Table 4 about here]

Table 4- Analogy between the hypotheses and an American perpetual call:

Option	option parameters and their effect on waiting to exercise the option				
American call	Equity volatility (+)	Exercise cost (+)	Exercise gain (−)	Stock's dividend (−)	Equity value (−)
real option to have debt	Asset volatility (+,H1)	Bankruptcy costs (+, H2)	Tax savings (−,H3)	Asset's payout (−,H4)	Unlevered asset value (−,H5)

Figure 4b also shows the ZL policy in the 2D plain of the volatility and bankruptcy costs: for a given bankruptcy cost, the firms move to the dark area across the vertical line as their volatility increases in the cross-section of volatility.[18] At the bankruptcy cost of 40%, a cut in the cross-section of the volatility yields a volatility boundary of 64%. Firms with asset volatility higher than the boundary stay in the dark area where it is feasible but not

[18]The natural question is "what are the general conditions for the waiting policy to exist?". Proposition 2 in the online appendix formally discusses the general criteria on the net debt benefit function for which there is value in waiting, even if the immediate issuance has positive net benefits. Case I is a special case of the general conditions. Theoretically, the real option idea is applicable to any trade-off model which meets the criteria. For example, it is also possible to extend the model to cases which shareholders split the default costs with debt holders in a bargaining game (the model is available in Online Appendix I.8). It does not change the hypotheses.

optimal to have debt. In the 1996-2015 period, 18% of the firm-quarters have asset volatility higher than 64% (see Section 3.1.1). *Ceteris paribus*, consider two firms with even the same unlevered values while one has riskier assets than the other. Having debt has added value for both firms. The low-risk firm issues debt right away because the gain is larger than the waiting option. The riskier firm prefers to wait because the option is more valuable than the immediate gain; when a firm is riskier, it is more likely to remain zero-leveraged. Hence, there is a boundary in the volatility cross-section of the firms. The boundary separates the firms that issue from the firms that do not issue based on the volatility of their assets. The model does not imply that every firm with a volatility higher than the boundary has zero leverage. It only applies to the firms that did not pass the boundary before. The boundary is not also constant and changes with the firm's situation. A firm issues debt under favorable circumstances; but later circumstances may change and the boundary drops, while the firm already has debt. For example, the borderline in Figure 4b represents all the volatility boundaries when the bankruptcy cost changes. A firm facing higher bankruptcy costs has a lower volatility boundary compared to another firm with lower bankruptcy costs. In Section 2.1.3, I analyze the boundary and develop the method to estimate the border line. Later, I estimate the borderline empirically in Section 3.2.3 with the form presented in Equation 13.

At 64% volatility, the traditional model without the real option only produces ZL policy with 90% default costs. Using the default cost estimates reported by Glover (2016), less than 5% of the firms face 90% bankruptcy cost.[19] Hence, this cost is way above the observed empirical values while the model with the real option only requires 40% bankruptcy cost to produce ZL policy. The comparison with the traditional model shows two more contributions of considering the real option in this case: a) the model with the real option demands more reasonable costs compared to the model without the real option b) the real option has a

[19] Glover (2016) reports an average cost of 45% with 27% standard deviation. Assuming a normal distribution for the cost shows that less than 5% of the observations may face default costs above 90%. The ratio seems even lower than 5% since his reported distribution has positive skewness.

first-order effect similar to the bankruptcy costs on ZL policy.

First, without the real option, most of the traditional models require inflated debt costs to show negative net benefits and explain zero leverage. For example, Sundaresan et al. (2015) point out the cost of lost growth options if the entrepreneur leaves the firm at default. Luciano and Nicodano (2014) examine the lost guarantees for the parent company guaranteeing subdivision's debt. Sufi (2009) finds that it is costly for some firms to have debt because of financial constraints to access debt markets. Studies in this literature mostly focus on showing hidden or ignored costs of debt. One strategy is adding other costs, such as debt overhang or family-legacy concerns. Another strategy is to consider constraints on issuing debt, also categorized as costs. Yet, including all these costs and constraints does not seem to exceed the tax savings for many ZL firms and the empirical works still highlight the puzzle (see Strebulaev and Yang (2013), Korteweg (2010), Bessler et al. (2013)). To reconcile both streams, I show that there is no need to inflate the ignored costs or hidden constraints. As long as the costs or constraints are non-convex and irreversible, they contribute to ZL policy. Within the range of the observed empirical values, the weaker assumption is enough to have zero leverage as the optimal strategy with positive issuance NPV. Second, the effect of the real option on ZL policy is similar to bankruptcy costs. For example, the option reduces the bankruptcy costs below half of the costs required by the traditional model to induce ZL policy.

2.1.3 Volatility and leverage relation

Another result of the model is about the kink that is created in the volatility-leverage relation due to the real option to have debt; the real-option model implies zero leverage for riskier firms above the volatility boundary, which creates a kink in the relation. For each model (traditional and the model with the real option) in the simulations, I rank the last 100 quarters of all the 1000 economies for each firm by their size into deciles. Then, I average their optimal leverage. For each firm there are 10 leverage data points which allows to substantially reduce the size of the sample into 5000 for each model. Every dot in Figure 6 represent a point for each firm.[20] The traditional model already predicts a negative relation between leverage and asset volatility but the average optimal leverage remains above 20%. For low volatility firms, both models almost create identical results. While the traditional model still considers optimal leverage to remain mostly above 10% for high volatility even as high as 90%, the model with the real option begins to generate optimal zero leverage for volatility higher than 40% depending on other parameters. Therefore, the real-option model deviates from the traditional model for riskier firms because the model with the real option implies zero leverage. Naively fitting a line to the model with the real option roughly shows the kink in the volatility-leverage relation. The kink is due to the volatility boundary. Above th boundary, leverage suddenly drops into zero and creates a breaking point.

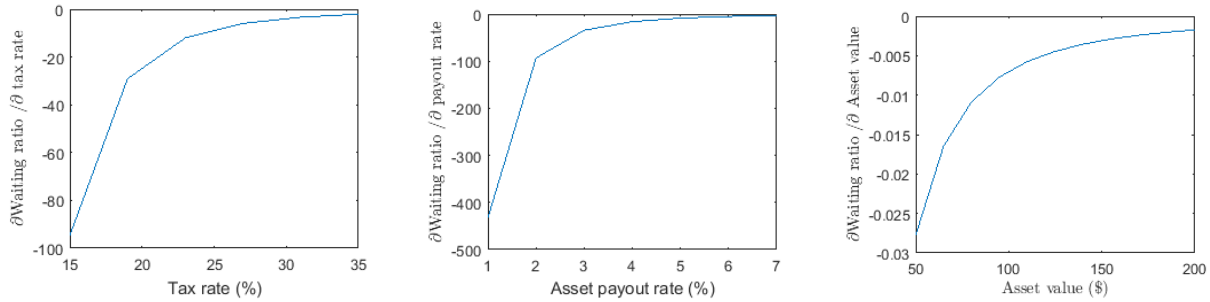
[Place Figure 6 about here]

Fitting the kinked line is not a very accurate method to estimate the breaking point because it ignores the effect of the other variables. A proper estimate of the boundary should be able to approximate the border line in the figures similar to Figure 4b. The following econometrical method uses re-interpreting PROBIT regression to provide a more accurate estimate. A general form of the PROBIT regression is:

$$
\begin{aligned}
&\Pr(\text{ZL}) = N\big[\Psi(\nu_0, log(\sigma), \Theta)\big] \\
&\Psi = d_0 + d_1 log(\sigma) + d_2\nu_0 + d\Theta \\
&\Theta = \{\text{other model parameters such as PBC rate } (\alpha)\text{, etc.}\}
\end{aligned}
\tag{6}
$$

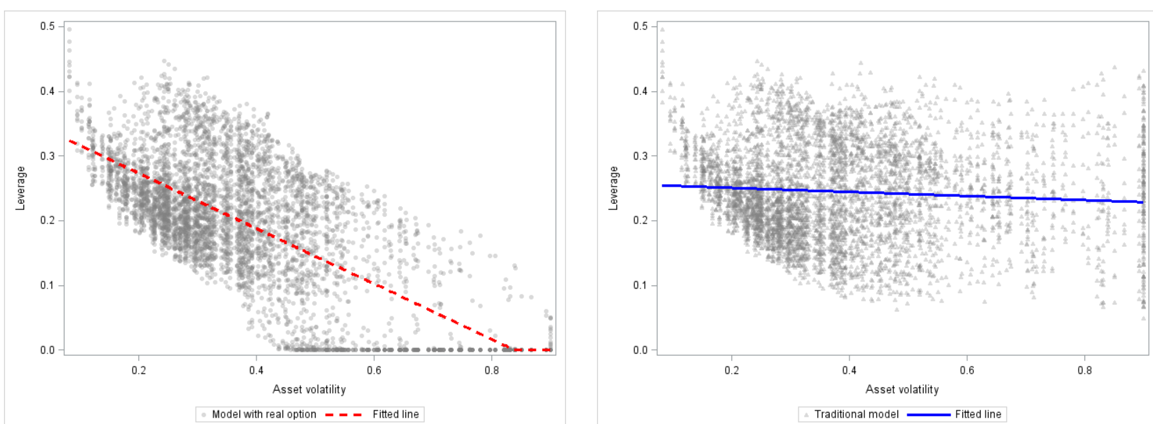
where $N[.]$ is standard cumulative normal distribution, d is regression coefficient and Ψ is the latent choice factor. Volatility appears with the log transformation in order to guarantee estimate of a positive boundary. Without log-transformation the boundary may take negative values which is mathematically correct as a lower bound but it is not informative. After estimating the model and assuming that the coefficients follow the earlier hypotheses, it

[20]The results are similar when I average all the observations for each firm rather than sorting them into deciles.



(a) **(b)** **(c)**

Figure 5. The effect of the independent variables on the waiting ratio in Case I: The X-axis shows the independent variable. The Y-axis shows the slope in the waiting ratios, $\frac{\partial(\nu_I/\nu_0)}{\partial x}$, where x is the independent variable. All the slopes are negative which support the hypotheses, H3, H4 and H5. In all the figures, the volatility is 0.6 and the bankruptcy cost rate is 40%. All other parameters are the same as Figure 4.



(a) The volatility-leverage relation in the simulation for the model with the real option.

(b) The volatility-leverage relation in the simulation for traditional model without the real option.

Figure 6. The optimal leverage in the simulated cross-section of volatility with and without the real option to have debt in Case I: The X-axis shows the firm's asset volatility. The Y-axis shows the optimal leverage. The draws in all the 1000 economies for the last 100 quarters of each firm are sorted into size deciles. The average optimal leverage of each decile is plotted in line with the asset volatility for each model. There are 5000 points for each model. Figure 6a which represents the model with the real option shows a kink in the line fitting the volatility-leverage relation in comparison to Figure 6b which represent the traditional model. The kink is due to the volatility boundary above which firms are very likely to have zero leverage and wait, even if the traditional model without the real option predicts that these firms should have leverage. For high volatility firms, comparing the figures shows that the optimal leverage in the model with the real option deviates from the traditional model. The simulation parameters are in Table 2.

follows:

$$\hat{ZL} = \begin{cases} 1 \quad \text{if} \quad N[\hat{\Psi}] > 0.5 \Leftrightarrow \hat{\Psi}(\nu_0, \sigma, \Theta) \geq 0 \Leftrightarrow \nu_0 \leq \hat{\nu_I}(\sigma, \Theta) \Leftrightarrow \sigma \geq \hat{\sigma_I}(\nu_0, \Theta), \\ 0 \quad \text{if} \quad Otherwise \end{cases} \tag{7}$$

Equation 7 is another representation of PROBIT that is equivalent to Equation 6. Rearranging the inequality yields:

$$ZL = 1 : \sigma \geq \hat{\sigma_I} = exp(-[\hat{d}_0 + \hat{d}_2\nu_0 + \hat{d}\Theta]/\hat{d}_1) \tag{8}$$

where σ_I is the borderline separating the cross-section of the firms into issuing and non-issuing firms similar to Figure 4b.[21] The exponential function which is the inverse of the log-transformation guarantees estimating a positive form for the boundary. I calculate the estimated boundary, $\hat{\sigma_I}$, for each data point and use the median to represent the break point in the volatility-leverage relation. In order to show the validity of the method, I test the method on a simple PROBIT regression and estimate the border line in Figure 4b and the kink in the volatility-leverage relation. I run the following regression on the first quarter of simulated data:

$$\begin{cases} \text{Pr(ZL)} = N\big[d_0 + d_1 log(\sigma) + d_2\alpha + d_3 m\big] \\ \text{ZL} = 1 : \sigma > \sigma_I = exp(-[d_0 + d_2\alpha + d_3 m]/d_1) \end{cases} \tag{9}$$

I use the proposed PROBIT equation re-arrangement and estimate the boundary for each data point. The results are in Figure 7 and Table 5. Figure 7 shows the estimated points and is comparable to Figure 4b. The median boundary is about 64% which also matches with the boundary reported in Figure 4b for the average bankruptcy cost.

[Place Table 5 about here]

[Place Figure 7 about here]

[21] There is no error term appearing in the final derivation because the error in PROBIT is the difference between the estimated probabilities, $\hat{e}_i = Pr(ZL_i) - \hat{N}(X_i\hat{\beta})$ where $Pr(ZL_i = 1) = 1$ and $Pr(ZL_i = 0) = 0$. For example, if $ZL_i = 0$, $\hat{N}(X_i\hat{\beta})$ should yield 0 for a perfect estimation.

Table 5- Boundary estimation parameters: I estimate $\Pr(\text{ZL}) = N\left[d_0 + d_1 log(\sigma) + d_2\alpha + d_3 m\right]$ on the first quarter of the simulated data with 500 data points. Volatility boundary is the median of the boundary for each data point calculated by using Equation 8.

Parameter	Obs.	Binary PROBIT regression coefficients d_0	d_1	d_2	d_3	median volatility boundary
estimate	500	-4.65	21.13	18.57	67.98	0.64
p-value	-	(0.39)	(0.01)	(0.03)	(0.19)	(0.01)

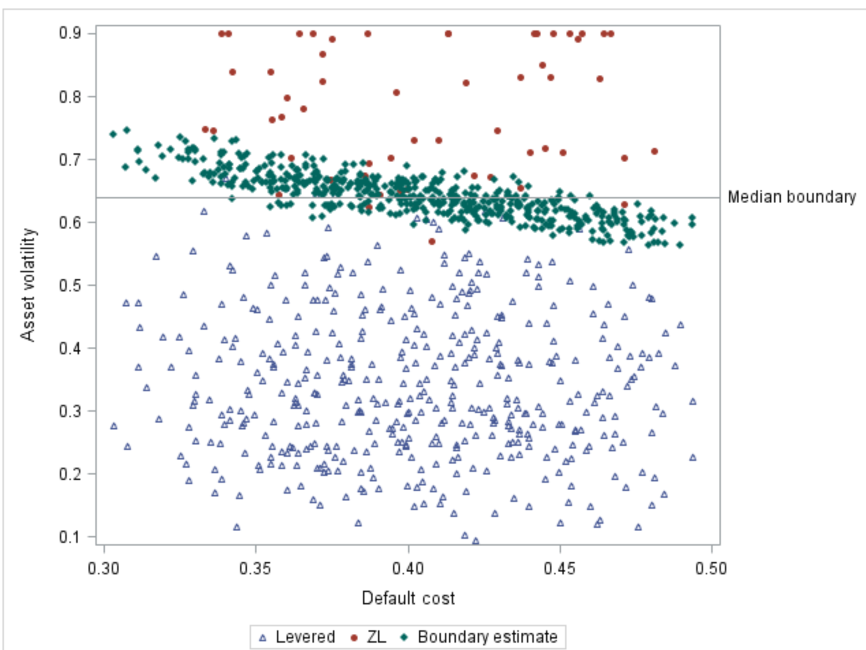


Figure 7. Volatility boundary estimation: The figure is empirical estimation of Figure 4b using the simulation data. The X-axis shows the bankruptcy cost rate. The Y-axis shows the asset volatility. Each point represent a firm's leverage decision along with the volatility and bankruptcy costs. The points with diamonds form the estimated boundary in Figure 4b. Each diamond point is created by applying the re-interpretation of PROBIT to Equation 9. The median represent the estimated kink in the leverage-volatility relation at 64% volatility.

The functional form for the volatility boundary, $\sigma_I = exp(.)$ in Equation 8, is an empirical equivalent to the borderlines between the dark and bright areas in Figures 4b. This re-interpretation of the choice regression in form of estimating the borderline is more intuitive because it matches with the theoretical implications. For example, H2 states that low bankruptcy costs decrease the ZL likelihood; in Figure 4b, the dark area shrinks and the borderline goes up. $\hat{\sigma_I}(\alpha, m)$ estimates the borderline in Figure 4b. Equivalent to H2, the estimated volatility boundary, $\hat{sigma}_I$, has negative relation with the bankruptcy costs because $-d_2/d_1$ is negative in Equation 9. A similar logic applies to the other hypotheses.

In Equation 7, the underlying choice factor, Ψ, is merely a statistical factor. It is compared with zero because zero is the de facto reference for the latent choice factor in PROBIT. However, rearranging the inequality yields more intuitive latent factors and reference points such as the volatility boundary, σ_I, or the issuance threshold, ν_I. In addition, the rearrangement in Equation 7 shows that a bijection (one-to-one correspondence) exists between the volatility boundary in the volatility cross-section and the threshold in the unlevered asset cross-section. It is trivial to mathematically show the bijection (see AppendixF). If the volatility boundary exists and one can estimate the boundary, then, the unlevered threshold to have debt also exists and one can also estimate its value. The equivalence is important because in econometric terms, I choose the volatility as the latent variable revealing the choice of the firm. There is no difference between estimating the volatility boundary or the unlevered threshold according to the bijection. Empirically, the estimation of the threshold for the firms' size does not provide a good sense about ZL strategy because it is in absolute terms and not relative. However, the unlevered asset volatility is already in relative terms, which allows comparing firms with respect to their asset volatility. Estimating volatility boundary is also simpler and in line with the real option intuition. Hence, this article only estimates the volatility boundary in re-interpreting PROBIT regressions.

3 Empirical Analysis

3.1 Data

I collect all the firm-quarters between 1996 and 2015 for the firms with no debt in the short or long term (Compustat codes DLTTQ and DLCQ) for at least one quarter. Following Strebulaev and Yang (2013), this article drops the utility and financial firms, non-US firms, and firms with an asset book value lower than $10 million. Each firm-quarter is flagged with a ZL indicator. Then, I match the firm-quarters with Optionmetrics database of 91-days call-option-implied volatilities for equity. I trim volatility below 0 and above 20. This is the main dataset of the paper and Table 6 has the descriptive statistics.

[Place Table 6 about here]

In this article, the ZL-firm criteria are similar to Strebulaev and Yang (2013) and Korteweg (2010). Both papers show that the firms matching their criteria benefit from having debt but mysteriously remain zero levered. Thus, relying on these studies, I use similar criteria and this article assumes that having debt is feasible for the firms in the main sample.[22]

There are two datasets in this article: a) the main dataset: the sample of the firm-quarters for the companies that have zero leverage at least in one quarter, and b) the second dataset: the sample of the all-time levered firms that never experienced zero leverage. The second dataset only serves for some anecdotal comparisons and the out-of-sample tests. The rest of the analyses, such as survival regression or re-interpreting PROBIT to estimate the boundary, exclusively focus on the main dataset.[23]

[22] In Compustat, there are also firms that may have almost zero leverage. Strebulaev and Yang (2013) define these firms to have a leverage below 5% which is smaller than the leverage produced by the dynamic models. There is stronger empirical support for Hypotheses 1 to 5 in the sample of the almost zero leverage firms as reported in Online Appendix I.2. In order to be parsimonious, this article limits itself to the pure ZL firms' sample to test the theory.

[23] I show in Online Appendix I.3 that the regression results are very similar when I pool both samples together to form the sample of all available firm-quarters between 1996 and 2015. Firms in the second sample never opted for zero leverage. Using them for estimating the regressions does not convey full information about the underlying choice. For example, it only provides a lower bound for the separating borderline shown in Figure 4b. The firms in the main dataset are the firms that hit the boundary. Thus, using the main sample yields a more accurate estimate of the regressions and the boundary. I use the out-of-sample tests to check how well using only ZL firms works in estimating PROBIT choice regression. I keep the second

Table 6- Descriptive statistics for the main sample of 1686 ZL firms in ZL and non-ZL firm-quarters: Data is from merged Compustat and Optionmetrics data for the 1996-2015 period. ZL (Zero-Leverage) is equal to 1 (yes) in a firm-quarter, if the firm is all-equity and long-term (DLTTQ) and short-term (DLCQ) debts are both zero and ZL is 0 (no) otherwise. Volatility is 91-day call-option-implied equity volatility. H volatility is 91-day historical volatility based on the daily stock prices. For non-ZL firm-quarters, volatility is delevered. Market cap is shares outstanding times the share price (CSHOQ×PRCCQ). Size is total debt plus the equity market cap ((DLTTQ + DLCQ)+ Market cap). Leverage is the total debt divided by size. Tangibility is the ratio of the book tangible assets to the total book assets ((PPENTQ/ATQ). Market value of the assets is the book liabilities (LTQ) plus the equity market cap. B/M ratio is the book assets (ATQ) to the market value of the assets. Profitability is the ratio of the firm's operating income before depreciation (OIBDPQ) to the market value of the assets. The tax ratio is the ratio of total income taxes to the net income or loss plus total income taxes(TXTQ/(NIQ+TXTQ)) . The payout rate is the dividend yield plus the share repurchase rate ((DVPSPQ/PRCCQ)+(PRSTKCY/Market cap)). Cash holdings is the ratio of cash and short-term investments to book assets (CHEQ/ATQ). Obs is the number of firm-quarter observations.

ZL	**Obs**	**Variable**	**Median**	**Mean**	**Std. dev.**	**5th %tile**	**95th %tile**
No	25733	Volatility	47.6%	53.4%	26.4%	22.4%	103.4%
		H volatility	44.5%	51.3%	30.1%	18.5%	108.1%
		Tangibility	12.3%	18.4%	18.2%	1.8%	59.6%
		B/M ratio	0.52	0.57	0.35	0.15	1.15
		Profitability	1.31%	0.80%	3.32%	-4.26%	4.10%
		Tax rate	27.9%	19.6%	39.5%	-14.5%	51.5%
		Payout rate	0.0%	5.0%	17.0%	0.0%	25.6%
		Size	714	3,118	17,070	84	10,008
		Cash/Asset	23%	29%	24%	1%	78%
Yes	23998	Volatility	54.8%	60.7%	27.2%	29.1%	111.5%
		H volatility	50.0%	57.3%	31.4%	23.9%	115.2%
		Tangibility	8.5%	13.7%	14.6%	0.9%	43.8%
		B/M ratio	0.42	0.49	0.33	0.12	1.07
		Profitability	0.95%	0.36%	3.38%	-4.78%	3.64%
		Tax rate	26.8%	18.4%	39.9%	-15.1%	49.6%
		Payout rate	0.0%	5.0%	16.9%	0.0%	24.6%
		Size	492	2,414	17,720	65	6,460
		Cash/Asset	41%	44%	24%	8%	89%

One of the important factors in the analyses is assets' volatility. For example, the volatility is used to re-interpret the choice regression. For ZL firm-quarters in the main sample, Optionmetrics' annualized option-implied equity volatility represents the assets' volatility because the firm is all-equity and debt-free. For non-ZL firm-quarters in the main sample, I delever the option-implied volatility using the market leverage ratio.[24] For the second dataset, the assets' volatility is also delevered option-implied volatility.

I limit data between 1996 and 2015 because data in Optionmetrics begins from 1996. Using the options' database does not substantially reduce the number of the firms in the main sample relative to the second sample in the 1996-2015 period. However, using option-implied volatility has two advantages to the prior studies. Bessler et al. (2013) use historical monthly equity volatility for ZL firms and delever it for the levered firms. Strebulaev and Yang (2013) use earnings volatility. First, the option-implied volatility is forward looking and reflects any changes in the market's beliefs that are not reflected in historical volatility. It is also instantaneous and does not need long time-series of data, compared to earnings volatility. I report historical volatility statistics in Table 6. The results are robust when I use historical equity volatility and they are not affected by the properties of option-implied volatility. I report the regression results with historical volatility in Online Appendix I.4. This article also transforms volatility with logarithm in the regressions to be able to estimate the volatility boundary, when it re-interpret the regression results. Without the log-transformation, it is possible to have a negative lower bound for the boundary in re-interpretation which is theoretically acceptable but it is not informative.

This article uses book-to-market (BM) and tangibility ratios to represent the bankruptcy

sample for the out-of-sample tests.

[24]Market leverage is QML, the ratio of book value of debt to the sum of equity's market value and the book value of debt. Volatility is delevered with $(1 - QML) \times volatility$ .Another method for delevering volatility is with the the total-liability-to-asset ratio. The other liabilities are relatively small for ZL firm-quarters compared to non-ZL firm-quarters. Using the liability ratio reduces the volatility for the non-ZL firm-quarters more than the ZL firm-quarters; it would strengthen the argument that the ZL firm-quarters have higher volatility. In order to remain parsimonious, this article delevers the volatility using the market leverage ratio. Operating leases, as an alternative source of debt, are not significantly different between ZL and levered firms. Including the leases does not have much effect on the sample with no explaining power over the choice of ZL policy similar to Strebulaev and Yang (2013).

costs. The tangibility is in book terms (PPENTQ/ATQ). I assume that the firms with higher tangible assets face lower bankruptcy costs. For another bankruptcy-cost proxy, I use the firms' growth opportunities indicated by the firm's BM ratio (ATQ / (Market cap+LTQ)) following the idea in Sundaresan et al. (2015): Consider two firms with all equal properties, including the unlevered value and tangibility while one has more growth opportunities than the other. The firm with high growth loses more at default because it loses more investment opportunities when the entrepreneur leaves the firm; the firms with low BM ratio have higher bankruptcy costs. *Ceteris paribus*, both BM and tangibility ratios test H2.

To measure effective tax savings, this article multiplies the tax rate by the profitability. The ratio of the firm's operating income before depreciation (OIBDPQ) to the market value of the firm (Market cap+LTQ) indicates the profitability. The tax rate is the ratio of total income taxes (TXTQ) to the net income plus taxes (NIQ+TXTQ). A similar measure based on the annual equivalent of TXTQ is used by Strebulaev and Yang (2013). I winsorize the tax ratio at the 5th percentiles because the ratio takes extreme values in some rare cases which the denominator is close to zero. The tax rate in the data is not volatile which causes the variable not to be significant by its own, but its effect is easier to detect through profitability. Moreover, highly profitable firms that face high tax rates are more likely to save taxes from having debt and less likely to follow ZL policy (H3). For H4, the regressions use the equity payout rate (the sum of dividend yield (DVPSPQ/PRCCQ) and share repurchase rate (PRSTKCY/Market cap)) and a dividend-paying dummy (1 if a firms has positive equity payout as dividend payer). Total debt plus the equity market cap ((DLTTQ + DLCQ)+ Market cap) is the total size of the firm with logarithm transformation that tests H5. In order to control for the firms seeking financial flexibility, this article includes the ratio of cash holdings and short-term investments (CHEQ/ATQ) to the ZL firms' book assets. High cash holdings imply high propensity for the firms to seek financial flexibility.

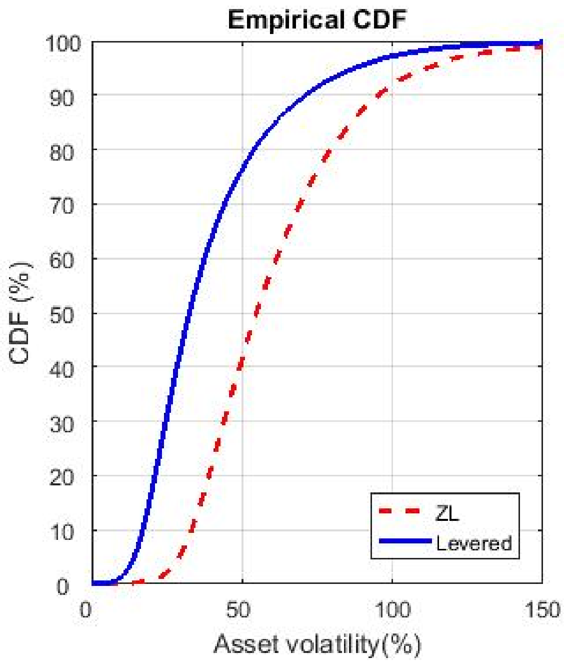
A naive look at the median and mean of the variables for ZL and non-ZL firm-quarters signals support for the hypotheses: the ZL firm-quarters have higher volatility (H1), higher bankruptcy cost proxies, i.e. low tangibility and BM ratio (H2), low tax ratio (H3), and are of smaller size (H5). The number of the ZL and non-ZL firm-quarters are close to each other. The balanced number of the firm-quarters in each category shows the better conditions to derive the boundary separating ZL and non-ZL firm-quarters in the ZL sample compared to the sample of all-time levered firms (see Footnote 23).

3.1.1 Asset volatility comparison

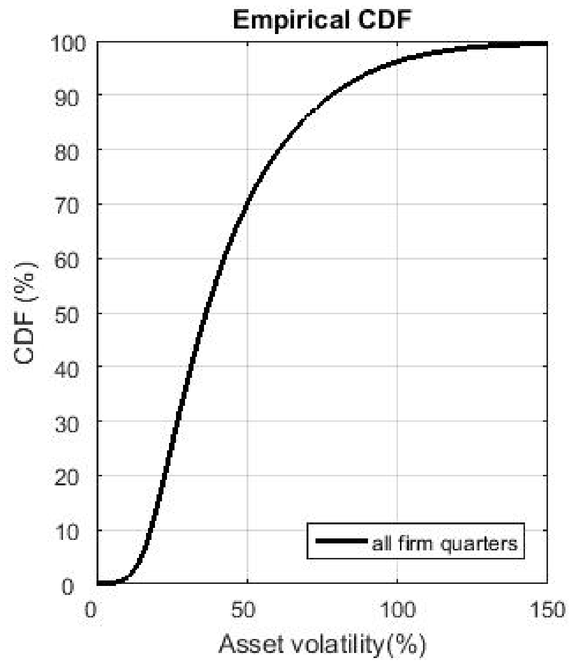
A volatility comparison between all the ZL and non-ZL firm-quarters between 1996 and 2015 shows that the ZL firm-quarters have higher asset volatility. The comparison is only a preliminary analysis and focuses on volatility because it relates to the real option intuition of the model. The distribution properties such as the average is also used in the simulations. Figure 8a shows the volatility cumulative probability distribution function (CDF) for two groups of firm-quarters: all ZL firm-quarters are compared to all levered firm-quarters in the 1996-2015 period. The ZL firm-quarters' average volatility is almost 1.5 times higher than the average for the levered firm-quarters in the same period. 87% of levered firm-quarters also fall below 64%, the average volatility boundary from the naive calibration of the model to the average parameters. Figure 8b shows the empirical CDF of the assets' volatility. Overall, 18% of all the firm-quarters have volatility higher than 64% between 1996 and 2015. The substantial differences in the volatility distributions and averages are support for H1.

On average, larger firms seem to have lower asset volatility. This relation may imply that the difference in volatility between the ZL and levered firm-quarters is created by their size difference. To address this issue, I put the firms in size deciles and show that the asset volatility is still higher for the ZL firm-quarters in the deciles. Figure 8c compares both averages of volatility and size in the deciles. The average volatility of the ZL firms is higher than their levered counterparts. The ZL deciles also have lower size compared to levered deciles. Again, this is anecdotal evidence for the real-option intuition.

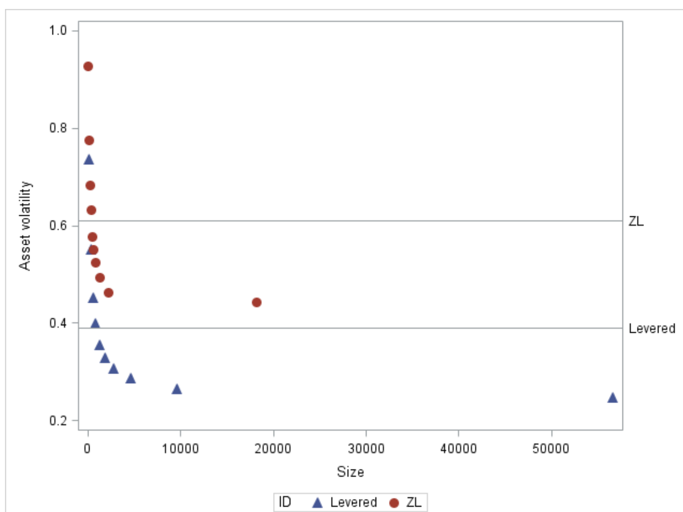
[Place Figure 8c about here]



(a) Empirical cumulative probability distribution function (CDF) comparison: The asset volatility of the ZL and the levered firm-quarters between 1996 and 2015.



(b) Empirical cumulative probability distribution function (CDF): The asset volatility of all the firm-quarters, ZL and levered, between 1996 and 2015.



(c) The volatility comparison between the size-sorted ZL and all-time levered firm-quarters: The X-axis indicates the average size. The Y-axis indicates the average asset volatility. For both groups, this article sorts the firm-quarters into size deciles. Each point represents the average for each decile during the 1996-2015 period. The triangles represent the levered firm-quarters. The circles represent ZL firm-quarters. The gray horizontal lines show the overall volatility averages without deciles.

Figure 8. Asset volatility comparison for the ZL and all-time levered firm-quarters: the preliminary analyses for H1

3.2 Regression Results

3.2.1 Survival analysis

The survival analysis tests the theory-implied hypotheses on the time that ZL firms remain debt-free and verifies the hypotheses. Between 1996 and 2015, I begin to follow each ZL firm when the first zero leverage quarter is recorded until the firm either issues debt for the first time or is censored from the sample. If a firm is censored, but later returns to the sample, I keep following it with a similar procedure. There are 1865 follow ups with 18,062 censored observations in the sample for 1686 firms. 920 firms experience issuing debt. The sample is skewed and unbalanced towards censored observations which represent ZL observations. The survival time is the quarters that a firm remains debt-free before taking on debt for the first time or is censored from the sample. The average consecutive quarters for which a firm stays in the sample is 10.17. In the survival terminology, issuing debt is a failure and the analysis looks at the factors that increase the survival duration before failure. The independent variables are the firm characteristics during each quarter that the firm is ZL. Dummies for the years, industry and fiscal quarters control the fixed effects and the seasonality. The regression also controls for the left-censored data because the observations do not cover the time that the firms follow ZL policy before 1996. The survival analysis uses the exponential distribution which estimates the following relation:[25]

$$\begin{aligned} log(\text{ZL duration}) = a_0 &+ a_1\log(\text{volatility}) + a_{2a}\text{Tangibility} + a_{2b}\text{BM ratio} \\ &+ b_1\text{Profitability} + b_2\text{Tax Ratio} + a_3\text{Profitability} \times \text{Tax Ratio} \\ &+ a_{4a}\text{Div. dummy} + a_{4b}\text{Payout Rate} + a_5\log(\text{size}) + b_3\text{cash ratio} \end{aligned} \tag{10}$$

[**Place Table 7 about here**]

There is evidence to support five hypotheses in Table 7: High volatility (H1), high bankruptcy costs (H2), low tax payments (H3), high payout rate (H4), and small size (H5)

[25] a_1 coefficient tests H1, a_{2a} and a_{2b} test H2, a_3 tests H3, a_{4a} and a_{4b} test H4, and a_5 tests H5.

Table 7- Survival analysis: It estimates $log(\text{ZL duration}) = a_0 + a_1\log(\text{volatility}) + a_{2a}\text{Tangibility} + a_{2b}\text{BM ratio} + b_1\text{Profitability} + b_2\text{Tax Ratio} + a_3\text{Profitability} \times \text{Tax Ratio} + a_{4a}\text{Div. dummy} + a_{4b}\text{Payout Rate} + a_5\log(\text{size}) + b_3 \text{ cash} + \textit{dummies}$. Dummies represent years, industries, and fiscal quarters. H1, H3, and H5 have support: a_1 is positive and significant as in H1, a_3 is negative and significant as in H3, and a_5 is negative and significant as in H5. a_{2a} is positive and significant as in H2 in the last regression only. As in H2 and H4, a_{2b}, and a_{4b} have the right sign but are not significant possibly due to low power. a_{4a} is not significant. The p-values test the null hypothesis that the coefficient is zero.

Parameter	Estimated Coefficients for each statistical regression					
Model	(1)	(2)	(3)	(4)	(5)	(6)
Intercept	3.81	4.09	4.03	4.16	1.43	1.97
p-value	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.00)
log(Volatility)	0.41	0.15	0.41	0.16	0.23	-0.1
p-value	(0.00)	(0.10)	(0.00)	(0.09)	(0.06)	(0.27)
Tangibility	-2.1	-1.38	-2.15	-1.44	-0.48	0.004
p-value	(0.00)	(0.00)	(0.00)	(0.00)	(0.14)	(0.98)
B/M ratio	-0.62	-0.49	-0.56	-0.44	-0.16	0.06
p-value	(0.00)	(0.00)	(0.00)	(0.00)	(0.20)	(0.62)
Profitability	-	-	1.89	1.72	3.03	3.28
p-value	-	-	(0.05)	(0.06)	(0.00)	(0.00)
Tax	-	-	-	-	0.08	0.15
p-value	-	-	-	-	(0.37)	(0.08)
Profit*Tax	-	-	-0.02	0.05	-4.5	-4.77
p-value	-	-	(1.00)	(0.99)	(0.05)	(0.03)
Div. Payer	-	-	-	-	0.28	0.31
p-value	-	-	-	-	(0.00)	(0.00)
Payout rate	-	-	-0.47	-0.54	-0.54	-0.65
p-value	-	-	(0.00)	(0.00)	(0.00)	(0.00)
log(Size)	-0.1	-0.08	-0.11	-0.09	-0.06	-0.05
p-value	(0.00)	(0.01)	(0.00)	(0.00)	(0.10)	(0.12)
Cash	-	-	-	-	3.35	3.04
p-value	-	-	-	-	(0.00)	(0.00)
Annual, industry, fiscal quarter dummies	yes	no	yes	no	yes	no
AIC	7371	7325	7360	7309	7093	7018

increase the time spent as ZL firm.[26]. Firms with high cash holdings and profitability spend more time having ZL, which seems coherent with the financial flexibility explanation. In sum, there is evidence in the survival regression to support the theoretical predictions.

3.2.2 PROBIT regression

The choice regression estimates the propensity to have no debt. In the PROBIT regression, I use the methodology recommended by Petersen (2009) and Gow, Ormazabal, and Taylor (2010) to control clustered time and industry errors while I also control fixed time and industry effects. PROBIT has the following form where $N[.]$ is standard cumulative normal distribution:

$$
\begin{aligned}
\Pr(\text{ZL}) = N\big[&a_0 + a_1 \log(\text{volatility}) + a_{2a}\text{Tangibility} + a_{2b}\text{BM ratio} \\
&+ b_1\text{Profitability} + b_2\text{Tax Ratio} + a_3\text{Profitability} \times \text{Tax Ratio} \\
&+ a_{4a}\text{Div. dummy} + a_{4b}\text{Payout Rate} + a_5\log(\text{size}) + b_3\ \text{cash} + \text{dummies}\big]
\end{aligned} \tag{11}
$$

Control dummies are year, industry, and fiscal quarter dummies. Table 8 shows the regression results. The coefficients related to the hypotheses are statistically significant and have the right sign.[27] The results support the theoretical model's predictions and the real-option intuition: higher volatility, higher debt costs, lower tax payments, lower payout rate and smaller size increase the likelihood of following ZL policy. Volatility, size, and bankruptcy costs solely contribute to more than half of the pseudo R-squared in the regression.

[Place Table 8 about here]

The regression results also contributes to the debate over the role of profitability in the leverage decisions. For the levered firms, Strebulaev (2007) provides a brief review on the dilemma in profit-leverage relationship: If the equity value for a levered profitable firm increases, constant debt level implies a decreasing leverage and negative profitability-

[26]The test seem to have low power because there is only 920 failure observations in the unbalanced sample (see also Footnote 27)

[27] In Appendix E, I show with simulations that fitting a linear line to a non-linear relation reduces the power of the test and some significant variables may look insignificant.

Table 8- PROBIT regression results : It estimates Pr(ZL) $= N[a_0 + a_1\log(\text{volatility}) + a_{2a}\text{Tangibility} + a_{2b}\text{BM ratio} + b_1\text{Profitability} + b_2\text{Tax Ratio} + a_3\text{Profitability}\times\text{Tax Ratio} + a_{4a}\text{Div. dummy} + a_{4b}\text{Payout Rate} + a_5\log(\text{size}) + b_3\text{ cash}] + dummies$. $N[.]$ is standard cumulative normal distribution. Dummies represent years, industries, and fiscal quarters. Expected signs are: a_1 to be positive and a_{2a}, a_{2b}, a_3, a_{4a}, a_{4b}, and a_5 to be negative. All the hypotheses are supported by the results: High volatility (a_1), high debt costs(a_{2a}, a_{2b}), low tax payments (a_3), low payout rate (a_{4b}) and small size (a_5) increase the propensity to remain ZL. Only a_{4a} that is related to H4 has a different sign. The p-values test the null hypothesis that the coefficient is zero.

Parameter	Estimated Coefficients for each statistical regression								
Model	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept	1.69	1.69	1.25	1.70	1.70	1.28	0.71	0.71	0.37
p-value	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
log volatility	0.46	0.46	0.25	0.47	0.47	0.27	0.40	0.40	0.20
p-value	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Tangibility	-1.33	-1.33	-0.82	-1.34	-1.34	-0.83	-0.34	-0.34	-0.16
p-value	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.11)	(0.00)
B/M ratio	-0.64	-0.64	-0.59	-0.65	-0.65	-0.61	-0.41	-0.41	-0.37
p-value	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Profitability	-	-	-	0.15	0.15	0.55	1.62	1.62	2.29
p-value	-	-	-	(0.52)	(0.76)	(0.02)	(0.00)	(0.03)	(0.00)
Tax rate	-	-	-	-	-	-	0.13	0.13	0.15
p-value	-	-	-	-	-	-	(0.00)	(0.00)	(0.00)
Profit*Tax	-	-	-	1.18	1.18	0.50	-2.14	-2.14	-2.89
p-value	-	-	-	(0.12)	(0.19)	(0.52)	(0.06)	(0.13)	(0.03)
Div. Pay. Dummy	-	-	-	-	-	-	0.27	0.27	0.31
p-value	-	-	-	-	-	-	(0.00)	(0.00)	(0.00)
Payout rate	-	-	-	0.16	0.16	0.23	-0.07	-0.07	-0.04
p-value	-	-	-	(0.00)	(0.09)	(0.00)	(0.13)	(0.43)	(0.24)
Log(Size)	-0.12	-0.12	-0.11	-0.12	-0.12	-0.11	-0.12	-0.12	-0.11
p-value	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Cash	-	-	-	-	-	-	1.66	1.66	1.35
p-value	-	-	-	-	-	-	(0.00)	(0.00)	(0.00)
Time, Industry, Fiscal Quarter dummies	yes	yes	no	yes	yes	no	yes	yes	no
Control clustered time and industry errors	no	yes	no	no	yes	no	no	yes	no
Pseudo R^2	21%	21%	8%	21%	21%	8%	26%	26%	14%

leverage relation. He also explains with simulations why the observed relation appears in the leverage regressions. His explanation relies on the unchanged debt levels in between of debt rebalancing points for the profitable firms. However, the explanation does not apply when the leverage is already zero because the leverage does not change with the variability in the equity's value. For ZL choice, the coefficient for profitability is positive while the interaction with the tax rate has the negative coefficient. Positive coefficient for profitability is more in line with pecking order while the negative coefficient for the interaction with taxes is more in line with trade-off theory. The result has implications for the financial policy in a profitable ZL firm: There are two separate channels that affect the decision to have debt. First is the pecking order channel where the firm prefers to use their internal funds and do not issue debt. The channel makes the profitability sign positive in the regression. Second is the tax channel, where the profitable firm is less likely to follow ZL policy, if it pays high taxes. Therefore, both pecking-order and trade-off channels can explain the relationship between profitability and the leverage in the ZL firms.

In the PROBIT regression, only the dividend dummy does not have the right sign because the dummy ignores the payout level. However, the payout rate has the right sign in the regressions. Tax ratio also has a positive sign but as soon as the tax ratio is multiplied by the profitability, the sign changes which supports H3. The following regressions also yield similar results but they are omitted for brevity: results do not change when the same regression is run cross-sectionally on data every year. The results are also robust to the error distribution when Logistic distribution (LOGIT) is used instead of the normal distribution.

3.2.3 Re-interpreting PROBIT regression

In order to re-interpret the regression, this article uses another representation of PROBIT that is equivalent to Equation 11:

$$\Psi = a_0 + a_1\text{log(volatility)} + a_{2a}\text{Tangibility} + \ldots + a_5\text{log(size)} + b_3 \text{ cash} + \textit{dummies},$$
$$ZL = \begin{cases} 1 \quad \text{if} \quad \Psi(\sigma, \Theta) \geq 0 \Leftrightarrow \sigma \geq \sigma_I(\Theta), \\ \\ 0 \quad \text{if} \quad \textit{Otherwise} \end{cases}$$
$$\Theta = \{\text{other model parameters such as size, PBC rate } (\alpha)\text{, etc.}\} \tag{12}$$

Rearranging the inequality yields:

$$ZL = 1 : \sigma \geq \sigma_I = exp(-[a_0 + a_{2a}\text{Tangibility} + \ldots + b_3 \text{ cash} + \textit{dummies}]/a_1) \tag{13}$$

where σ_I is the volatility boundary separating the cross-section of the firms into issuing and non-issuing firms similar to Figure 4b in the theoretical model. In the main sample, I run the steps described in Section 2.1.3 on Model (8) from Table 8 as the model has all the independent variables. The median estimated volatility boundary for the ZL firm-quarters is 30%, below their volatility median of 55% ($\sigma^{ZL} = 55\% \geq 30\% = \sigma_I^{ZL}$). The boundary median is 104% for the firm-quarters with leverage, above their volatility median of 48% ($\sigma^{non-ZL} = 48\% \leq 104\% = \sigma_I^{non-ZL}$). Theoretically, it is equivalent to the ZL firm-quarters located above the volatility boundary in Figure 4b and non-ZL firm-quarters are located far below. In addition, the boundary is higher when the firms decide to take on debt compared to the boundary when they follow ZL ($\sigma_I^{non-ZL} = 104\% \geq 30\% = \sigma_I^{ZL}$). It is due to the changes in the other factors, such as size and debt costs, which create more favorable conditions to have leverage. Using both ZL and non-ZL firm-quarters, the estimated overall volatility boundary is about 54%. The 54% boundary is also close to 64%, the estimated boundary in the model calibrations. Anecdotally and without considering any other factors such as size or bankruptcy costs, a firm with an asset volatility above 54% may consider the real option to have debt and remain ZL.[28] Therefore, in addition to re-interpreting the statistical model in line with the model intuition, estimating the boundary yields a reasonable value above which debt-free capital structure is justifiable.

3.3 Robustness check

3.3.1 Cross-sectional out-of-sample test

The out-of-sample (OOS) tests check how well the PROBIT regression extends cross-sectionally to the other firms. The results validate the regression because the choice regression is able to reasonably predict the behavior of the all-time levered firms.Bessler et al. (2013) check the PROBIT with OOS test in predicting ZL choice over time, but this article checks the prediction power in a blind cross-sectional OOS test. The test is analogous to using the factors that determine the unemployment decision on predicting the behavior of the people who decided to be employed. Table 9 presents the results. The table reports the overall prediction error of the choice regression. To create the table, I run the PROBIT regression on the firm-quarter sample of ZL firms that have ZL at least once between 1996 and 2009. Then, I use the coefficients to estimate the choice of the all-time levered firms which never have ZL in the firm-quarters between 2010 and 2015. I drop any firms in the levered sample that would match with the first sample to assure that the test is a blind out-of-sample test. The error rate in the OOS test is the ratio of the firm-quarters predicted to have zero leverage to the sample size. A perfect PROBIT regression predicts that the firm-quarters in the second sample will not choose the ZL policy at all. The PROBIT seems to do reasonable predictions about the all-time levered firms. The OOS test also measures the contribution of considering the assets' volatility. The marginal contribution of having the volatility in the PROBIT regression to reduce the prediction error is about 8%, which

[28] 25% of all the firm-quarters, levered and ZL, between 1996 and 2015 have a volatility higher than 54%, which is close to the rate of ZL behavior observation among the firms.

is close to 2,102 firm-quarters. The volatility's contribution supports H1 which is also an important intuition behind the real-option model.

[Place Table 9 about here]

Table 9- Out-of-sample (OOS) test errors : It cross-sectionally tests how well using only ZL firms to estimate the PROBIT regression can predict the leverage choice of all-time levered firms. The choice regression is estimated in the 1996-2009 period and predicts the leverage choice of all-time levered firms in 2010-2015 period. The first regression on the left with all the the variables is Model (9) from Table 8. The next regressions gradually lose some of the independent variables to show their predictive contribution. The second regression is without cash and profitability ratios. The third regression is similar to the second without the payout dummy and payout ratio. The fourth regression only has asset volatility and size and the last has only the size variable. All the regressions have an intercept. Errors are the ratio of the firm-quarters predicted to have zero leverage by the choice regression to the sample size.

Out-of-sample test	Obs.	Binary PROBIT regression models with				
		all variables	No cash and profitability	No payout, cash and profitability	Only size and volatility	only size
Error	26270	6.78%	9.00%	8.10%	8.13%	16.96%

3.3.2 The subsamples: zero-interest proxy and the firms with equity payout

This article reports the robustness in results for at least two subsamples of the main sample: a) the firms that do not pay any interest, as another ZL proxy, and b) the firms that have payouts to equity holders. In Appendix H, I also show that the inferences are robust to controlling for governance factors. Table 10 shows all the results similar to the earlier PROBIT with higher R-squared. The earlier PROBIT regressions use the definition proposed in Strebulaev and Yang (2013) for the ZL firm-quarters, which is about the firms without any long or short term debt. An alternative proxy is the ZL firm-quarters that also pay zero interest (ZI) on their balance sheet. ZI firm-quarters are a subsample within the ZL firm-quarters. Using the new proxy makes the inferences stronger. The profitability is also less significant contrary to the earlier estimations. In the profitability-leverage relationship, the result implies that the ZI firms prefer to have no interest-bearing obligations because they are more concerned about hedging bankruptcy costs. Therefore, the bankruptcy hedging channel seems more important to the ZI firms than the pecking order channel.

[Place Table 10 about here]

Table 10- PROBIT regression results on the subsamples of the ZL firms: Subsample (1) has only firm-quarters with payout to shareholders. In Subsample (2), ZL proxy is the firms with no interest payments; ZL is 1, if the firm has zero leverage in at least one quarter and does not pay any interest. The regressions estimate $\Pr(\text{ZL}) = N[a_0 + a_1\log(\text{volatility}) + a_{2a}\text{Tangibility} + a_{2b}\text{BM ratio} + b_1\text{Profitability} + b_2\text{Tax Ratio} + a_3\text{Profitability} \times \text{Tax Ratio} + a_{4a}\text{Div. dummy} + a_{4b}\text{Payout Rate} + a_5\log(\text{size}) + b_3\ \text{cash} + \textit{dummies}]$. $N[.]$ is standard cumulative normal distribution. Dummies represent years, industries, and fiscal quarters. Expected signs are: a_1 to be positive and a_{2a}, a_{2b}, a_3, a_{4a}, a_{4b}, and a_5 to be negative. All the hypotheses are supported by the results: High volatility (a_1), high debt costs(a_{2a}, a_{2b}), low tax payments (a_3), low payout rate (a_{4b}) and small size (a_5) increase the propensity to remain ZL. Only a_{4a} that is related to H4 has a different sign in Subsample (2). a_{4b} is not significant in Subsample (1). The p-values test the null hypothesis that the coefficient is zero.

Parameter	Estimated Coefficients for each statistical regression			
	Firm-quarters with payout		Firm-quarters with no interest payments	
Model	(1)	(2)	(3)	(4)
Obs.	21,912	21,912	26,619	26,619
Intercept	1.65	1.65	0.58	0.58
p-value	(0.00)	(0.00)	(0.00)	(0.01)
log volatility	0.43	0.43	0.22	0.22
p-value	(0.00)	(0.00)	(0.00)	(0.02)
Tangibility	-0.67	-0.67	-0.58	-0.58
p-value	(0.00)	(0.05)	(0.00)	(0.03)
B/M ratio	-0.84	-0.84	-0.42	-0.42
p-value	(0.00)	(0.00)	(0.00)	(0.00)
Profitability	1.49	1.49	0.35	0.35
p-value	(0.01)	(0.10)	(0.28)	(0.69)
Tax rate	0.05	0.05	0.12	0.12
p-value	(0.09)	(0.16)	(0.00)	(0.00)
Profit*Tax	-2.88	-2.88	-0.27	-0.27
p-value	(0.08)	(0.20)	(0.86)	(0.88)
Div. Pay. Dummy	-	-	0.23	0.23
p-value	-	-	(0.00)	(0.00)
Payout rate	0.03	0.03	-0.14	-0.14
p-value	(0.42)	(0.69)	(0.06)	(0.08)
Log(Size)	-0.17	-0.17	-0.09	-0.09
p-value	(0.00)	(0.00)	(0.00)	(0.00)
Cash	1.64	1.64	1.23	1.23
p-value	(0.00)	(0.00)	(0.00)	(0.00)
Time, Industry, Fiscal Quarter dummies	yes	yes	yes	yes
Control clustered time and industry errors	no	yes	no	yes
Pseudo R^2	33.6%	33.6%	23.4%	23.4%

The inferences are also robust in the subsamples of the ZL firms that additionally have positive payout, either in the form of dividends or share repurchases. These firms are more likely to be financially flexible and show the most puzzling behavior among the ZL firms. Not only they seem to ignore losing their financial flexibility, but also they tend to pay higher taxes when they replace interest payments with dividends. The factors that the theoretical model predicts to have effect on ZL policy have 1.5 times more explanatory power in this subsample. Thus, the real option intuition is empirically supported in the sample of dividend-paying ZL firms and complements the financial flexibility theory.

4 Conclusion and Future Research

This article shows that the debt-free firms do not lose value by not levering up because they optimally hold the real option to have debt later. The behavior is optimal even when the traditional trade-off models expect them to have leverage. I calculate the theoretical value in waiting to have debt. The value is more likely to exceed the immediate positive gain from having debt for small, and risky firms. As I show in the simulations, even with a positive net gain, managers prefer zero leverage and hold the real option in order to hedge exposure to bankruptcy costs. The value of the zero-leverage firm includes the real-option component and the real option suggests a new mechanism to explaining the zero-leverage puzzle. Therefore, I extend the inaction area from real-option literature, e.g Bloom (2009), to the trade-off and capital structure theory.

This article contributes to the studies on the zero-leverage phenomenon. Some earlier studies look for the debt-related costs to make the net debt benefits negative (e.g. Luciano and Nicodano (2014), Sundaresan et al. (2015), Sufi (2009)). However, empirical studies find that the benefits are still positive and the firms seem to leave the benefits on the table, e.g. Strebulaev and Yang (2013). I show that zero leverage and positive immediate net gains from having debt can coexist. As long as the debt costs are irreversible and non-convex, they contribute to ZL behavior through the real option to have debt. The condition is a weaker condition compared to requiring the costs to be larger than the tax savings. I also show that dividend payments and zero leverage are not contradictory. In this sense, this article complements the financial flexibility literature which faces challenges to explain dividend-paying zero-leverage firms.

Empirically, this article finds support for its theoretical predictions. While I control for other parameters such as cash holdings and profitability, I find the factors that explain the time to remain ZL. The estimation of the survival and choice regressions for the firms with zero leverage shows that most of the factors determined theoretically increase the propensity and duration to stay debt-free: High asset volatility, high debt costs, low tax payments, low payout rate, and small size. The out-of-sample test validates the empirical regressions. The findings are also robust in the subsamples and controlling for the governance.

An extension of this article for future can consider the relation between ZL and priced volatility risk with stochastic volatility. Another extension for future research can mix project inception and debt financing similar to Sundaresan and Wang (2007) and Tserlukevich (2008): financing choices between debt and equity for large versus small projects, and high-risk versus low-risk projects. The idea relates to the seemingly puzzling behavior of the firms in financing large projects with leverage according to Byoun, Kim, and Yoo (2013).

References

Anderson, R., and S. Sundaresan. 2000. A comparative study of structural models of corporate bond yields: An exploratory investigation. *Journal of Banking and Finance* 24:255 – 269. URL http://www.sciencedirect.com/science/article/pii/S037842669900059X.

Bessler, W., W. Drobetz, R. Haller, and I. Meier. 2013. The international zero-leverage phenomenon. *Journal of Corporate Finance* 23:196 – 221. URL http://www.sciencedirect.com/science/article/pii/S0929119913000783.

Bhamra, H. S., L.-A. Kuehn, and I. A. Strebulaev. 2010. The Aggregate Dynamics of Capital Structure and Macroeconomic Risk. *Review of Financial Studies* 23:4187–4241. URL http://rfs.oxfordjournals.org/content/23/12/4187.abstract.

Bloom, N. 2009. The impact of uncertainty shocks. *Econometrica* 77:623–685.

Bris, A., I. Welch, and N. Zhu. 2006. The costs of bankruptcy: Chapter 7 liquidation versus Chapter 11 reorganization. *Journal of Finance* 61:1253–1303.

Broadie, M., M. Chernov, and S. Sundaresan. 2007. Optimal debt and equity values in the presence of Chapter 7 and Chapter 11. *Journal of Finance* 62:1341–1377.

Byoun, S., J. Kim, and S. S. Yoo. 2013. Risk management with leverage: Evidence from project finance. *Journal of Financial and Quantitative Analysis* 48:549–577.

Dai, Z., D. A. Shackelford, H. H. Zhang, and C. Chen. 2013. Does Financial Constraint Affect the Relation between Shareholder Taxes and the Cost of Equity Capital? *Accounting Review* 88:1603–1627.

DeAngelo, H., L. DeAngelo, and T. M. Whited. 2011. Capital structure dynamics and transitory debt. *Journal of Financial Economics* 99:235 – 261. URL http://www.sciencedirect.com/science/article/pii/S0304405X10002175.

Denis, D. J. 2012. The Persistent Puzzle of Corporate Capital Structure: Current Challenges and New Directions. *Financial Review* 47:631–643.

Dixit, A. K., and R. S. Pindyck. 2012. *Investment Under Uncertainty.* [E-reader version]. Princeton University Press. URL http://www.jstor.org/stable/j.ctt7sncv.

Dotan, A., and S. A. Ravid. 1985. On the Interaction of Real and Financial Decisions of the Firm Under Uncertainty. *Journal of Finance* 40:pp. 501–517. URL http://www.jstor.org/stable/2327897.

Elkamhi, R., J. Ericsson, and C. A. Parsons. 2012. The cost and timing of financial distress. *Journal of Financial Economics* 105:62–81.

Ericsson, J., and J. Reneby. 1998. A framework for valuing corporate securities. *Applied Mathematical Finance* 5:143–163.

Fischer, E. O., R. Heinkel, and J. Zechner. 1989. Dynamic capital structure choice: Theory and tests. *Journal of Finance* 44:19–40.

Gamba, A., and A. Triantis. 2008. The value of financial flexibility. *Journal of Finance* 63:2263–2296.

Gilson, S. C. 1997. Transactions costs and capital structure choice: Evidence from financially distressed firms. *Journal of Finance* 52:161–196.

Glover, B. 2016. The expected cost of default. *Journal of Financial Economics* 119:284–299.

Goldstein, R., N. Ju, and H. Leland. 2001. An EBIT-Based Model of Dynamic Capital Structure. *Journal of Business* 74:483–512.

Gompers, P., J. Ishii, and A. Metrick. 2003. Corporate Governance and Equity Prices. *Quarterly Journal of Economics* 118:107–156. URL http://qje.oxfordjournals.org/content/118/1/107.abstract.

Gorbenko, A. S., and I. A. Strebulaev. 2010. Temporary versus Permanent Shocks: Explaining Corporate Financial Policies. *Review of Financial Studies* 23:2591–2647. URL http://rfs.oxfordjournals.org/content/23/7/2591.abstract.

Gow, I. D., G. Ormazabal, and D. J. Taylor. 2010. Correcting for cross-sectional and time-series dependence in accounting research. *Accounting Review* 85:483–512.

Graham, J. R., and C. R. Harvey. 2001. The theory and practice of corporate finance: Evidence from the field. *Journal of Financial Economics* 60:187–243.

Hackbarth, D., C. A. Hennessy, and H. E. Leland. 2007. Can the trade-off theory explain debt structure? *Review of Financial Studies* 20:1389–1428.

Hackbarth, D., and D. C. Mauer. 2012. Optimal priority structure, capital structure, and investment. *Review of Financial Studies* 25:747–796.

Hennessy, C. A., and T. M. Whited. 2005. Debt dynamics. *Journal of Finance* 60:1129–1165.

Kisser, M. 2013. The real option value of cash. *Review of Finance* 17:1649–1697.

Korteweg, A. 2010. The net benefits to leverage. *Journal of Finance* 65:2137–2170.

Leland, H. E. 1994. Corporate debt value, bond covenants, and optimal capital structure. *Journal of Finance* 49:1213–1252.

Leland, H. E. 1998. Agency costs, risk management, and capital structure. *Journal of Finance* 53:1213–1243.

Leland, H. E., and K. B. Toft. 1996. Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads. *Journal of Finance* 51:987–1019.

Luciano, E., and G. Nicodano. 2014. Guarantees, leverage, and taxes. *Review of Financial Studies* 27:2736–2772.

Mao, L., and Y. Tserlukevich. 2014. Repurchasing debt. *Management Science* .

McDonald, R., and D. Siegel. 1986. The value of waiting to invest. *Quarterly Journal of Economics* 101:707–727.

Modigliani, F., and M. H. Miller. 1958. The cost of capital, corporation finance and the theory of investment. *American Economic Review* 48:261–297.

Petersen, M. A. 2009. Estimating Standard Errors in Finance Panel Data Sets: Comparing Approaches. *Review of Financial Studies* 22:435–480. URL `http://rfs.oxfordjournals.org/content/22/1/435.abstract`.

Schaefer, S. M., and I. A. Strebulaev. 2005. Structural Models of Credit Risk are Useful: Evidence from Hedge Ratios on Corporate Bonds. *Working paper, London Business School [Later version published in Journal of Financial Economics, 2008]* .

Scherr, F. C., and H. M. Hulburt. 2001. The debt maturity structure of small firms. *Financial Management* pp. 85–111.

Strebulaev, I. A. 2007. Do tests of capital structure theory mean what they say? *Journal of Finance* 62:1747–1787.

Strebulaev, I. A., and B. Yang. 2013. The mystery of zero-leverage firms. *Journal of Financial Economics* 109:1–23.

Sufi, A. 2009. Bank Lines of Credit in Corporate Finance: An Empirical Analysis. *Review of Financial Studies* 22:1057–1088. URL `http://rfs.oxfordjournals.org/content/22/3/1057.abstract`.

Sundaresan, S., and N. Wang. 2007. Investment under uncertainty with strategic debt service. *American Economic Review* 97:256–261.

Sundaresan, S., N. Wang, and J. Yang. 2015. Dynamic investment, capital structure, and debt overhang. *Review of Corporate Finance Studies* 4:1–42.

Trigeorgis, L. 1993. Real options and interactions with financial flexibility. *Financial Management* 22:202–224.

Tserlukevich, Y. 2008. Can real options explain financing behavior? *Journal of Financial Economics* 89:232 – 252. URL `http://www.sciencedirect.com/science/article/pii/S0304405X08000925`.

Appendices

A Derivation of the unlevered asset value

The EBIT process, γ, under the RA measure is GBM. ω is the constant drift and smaller than risk-free rate, r. Unlevered asset value is the present value of all the future cash flows and using Ito's lemma it follows (as $E^q(\gamma_s|\gamma_t) = \gamma_t e^{\omega(s-t)}$):

$$\frac{d\gamma}{\gamma} = \omega dt + \sigma dW^q : \nu = E^q\Big(\int_t^\infty (1-\tau)e^{-r(s-t)}\gamma_s ds\Big) = \frac{(1-\tau)\gamma}{r-\omega} \Rightarrow \frac{d\nu}{\nu} = \omega dt + \sigma dW^q \quad (14)$$

B List of the variables and their description

See Table 11.

[Place Table 11 about here]

Table 11-List of the variables in the models and their descriptions

Variable	Description	Variable	Description
ν	Unlevered assets' value	r	Risk-free interest rate
μ	Historical drift	$\mu - r$	Asset risk premium
τ	Tax rate	δ	Assets' leak or payout rate
TS	Interest tax savings	DC	Debt costs
DB	Total net debt benefits	W	The real-option's value
F	Contingent claim's value	P	Face value of the outstanding debt
C	Coupon rate	C	Outstanding coupon rate
D	Debt value	E	Equity value
κ	Fixed debt rebalancing and issuance cost	F_ν	first-order derivative of F with respect to ν
k_1	Proportional debt rebalancing and issuance costs	k_2	Proportional transaction cost for debt rollover
M	Average debt maturity	K	Fixed bankruptcy cost
ν_I	The unlevered threshold, the optimal threshold for the unlevered assets' value above which the firms issue debt in the unlevered value cross-section		
ν_U	The rebalancing point, the optimal point for the unlevered assets' value at which the firm rebalances debt.		
ν_B	The default barrier, The optimal barrier for the unlevered assets' value below which the firm files for bankruptcy		
σ	Asset volatility, The standard deviation for the return on the unlevered assets' value		
σ_I	The volatility boundary, the boundary below which the firms issue debt in the volatility cross-section		
σ_{max}	The maximum volatility below which the net benefits from issuing debt are positive and the recapitalization is feasible		
α	Proportional Bankruptcy Cost (PBC) rate as the percentage of the unlevered value at the time of default		
dW^x	Standard Brownian shocks: $x = q$ under risk-adjusted measure, $x = p$ under physical measure, $x = O$ for idiosyncratic risk, $x = S$ for systematic risk.		

C Derivation of the formulas in Case I

Prior to showing the details, the summary of the derivations are presented below:

$$\nu_B < \nu < \nu_U \begin{cases} D(\nu) = \dfrac{C+mP}{r+m} + A_D\nu^{y_1} + B_D\nu^{-y_2}, \\ TS(\nu) = \dfrac{\tau C}{r} + A_T\nu^{\beta_1} + B_T\nu^{-\beta_2}, \qquad BC(\nu) = A_B\nu^{\beta_1} + B_B\nu^{-\beta_2} \\ RC(\nu) = k_1P + \dfrac{k_2mP}{r} + \kappa + A_R\nu^{\beta_1} + B_R\nu^{-\beta_2} \\ DC(\nu) = BC(\nu) - RC(\nu), \quad DB(\nu) = TS(\nu) - DC(\nu), \\ E(\nu) = \nu + DB(\nu) - D(\nu) \end{cases}$$
$$\nu \leq \nu_I, \text{(before issuance)}: \qquad W(\nu) = DB(\nu_I)(\frac{\nu}{\nu_I})^{\beta_1} \tag{15}$$

With debt retirement rate, m, the face value of debt declines as $dp = -mp(s)ds$. Hence, the face value is $p_s = p_t e^{-m(s-t)}$. The average debt maturity follows as:

$$M = \int_t^\infty (s-t)\frac{mp(s)}{p_t}ds = \int_t^\infty (s-t)\frac{me^{-m(s-t)}p_t}{p_t}ds = 1/m \tag{16}$$

At any point in time the firm retires mP part of outstanding debt so the face value of outstanding debt always remains constant for the firm:

$$\int_t^\infty mP(s)ds = \int_t^\infty mP_t e^{-m(s-t)}ds = P_t = P \tag{17}$$

Debt, D, equity, E, tax savings, TS, and debt costs, DC (bankruptcy, BC, and rebalancing costs, RC) are all claims defined on the unlevered asset value. In general, any perpetual claim on the unlevered value with continuous payments, G, satisfies the following PDE:

$$\frac{1}{2}\sigma^2\nu^2F_{\nu\nu} + (r-\delta)\nu F_\nu - rF + G = 0 \tag{18}$$

where F is the claim's value. This PDE is similar to the Black-Scholes PDE but it is used to value debt and other securities *post* issuance. The general solution is:

$$F = A_0 + A_1\nu^{\beta_1} + A_2\nu^{-\beta_2} \qquad \beta_2 = \frac{\sqrt{h^2+2r}+h}{\sigma} \tag{19}$$

where A_0, A_1, and A_2 depend on the Dirichlet conditions. β_1 is in Equation 3. A PDE similar to Equation 18, and solutions similar to Equation 19 apply to bankruptcy cost, rebalancing cost, and tax savings. The Dirichlet conditions for all $\nu_B < \nu < \nu_U$ are:

$$TS(\nu_B) = 0, \quad TS(\nu_U) = \phi TS(\nu_0), \quad TS(\nu) = \frac{\tau C}{r} + A_T\nu^{\beta_1} + B_T\nu^{-\beta_2} \tag{20}$$

$$BC(\nu_B) = \alpha\nu_B, \quad BC(\nu_U) = \phi BC(\nu_0), \quad BC(\nu) = A_B\nu^{\beta_1} + B_B\nu^{-\beta_2} \tag{21}$$

$$\begin{gathered} RC(\nu_B) = 0, \quad RC(\nu_U) = \phi RC(\nu_0), \quad RC(\nu) = \frac{k_2 mP}{r} + A_R\nu^{\beta_1} + B_R\nu^{-\beta_2} \\ RC(\nu_0) = k_1 P + \kappa + \frac{k_2 mP}{r} + A_R\nu_0^{\beta_1} + B_R\nu_0^{-\beta_2} \end{gathered} \tag{22}$$

$$DC(\nu) = BC(\nu) + RC(\nu), \quad DB(\nu) = TS(\nu) - DC(\nu) \tag{23}$$

Following the Dirichlet conditions, coefficient values for A_i and B_i are:

$$CM = \begin{bmatrix} \nu_B^{\beta_1} & \nu_B^{-\beta_2} \\ \nu_U^{\beta_1} - \phi\nu_0^{\beta_1} & \nu_U^{-\beta_2} - \phi\nu_0^{-\beta_2} \end{bmatrix}^{-1} \tag{24}$$

$$\begin{bmatrix} A_T \\ B_T \end{bmatrix} = CM \times \begin{bmatrix} -\tau C/r \\ (\phi-1)\tau C/r \end{bmatrix} \tag{25}$$

$$\begin{bmatrix} A_B \\ B_B \end{bmatrix} = CM \times \begin{bmatrix} \alpha\nu_B \\ 0 \end{bmatrix} \tag{26}$$

$$\begin{bmatrix} A_R \\ B_R \end{bmatrix} = CM \times \begin{bmatrix} -k_2 mP/r \\ \phi(\kappa + k_1 P) + (\phi-1)k_2 mP/r \end{bmatrix} \tag{27}$$

The debt value changes slightly as it is also dependent on the debt retirement rate, m:

$$\frac{1}{2}\sigma^2\nu^2 D_{\nu\nu} + (r-\delta)\nu D_\nu - (r+m)D + (C+mP) = 0 \tag{28}$$

$$\begin{cases} D(\nu) = \dfrac{C+mP}{r+m} + A_D\nu^{y_1} + B_D\nu^{-y_2}, \\ \\ y_1 = \dfrac{\sqrt{h^2+2(r+m)}-h}{\sigma} \quad y_2 = \dfrac{\sqrt{h^2+2(r+m)}+h}{\sigma} \end{cases} \tag{29}$$

The Dirichlet condition and the solution for debt are:

$$\begin{aligned} & D(\nu_B) = (1-\alpha)\nu_B, \quad D(\nu_U) = P, \\ & \begin{bmatrix} A_D \\ B_D \end{bmatrix} = \begin{bmatrix} \nu_B^{y_1} & \nu_B^{-y_2} \\ \nu_U^{y_1} & \nu_U^{-y_2} \end{bmatrix}^{-1} \times \begin{bmatrix} (1-\alpha)\nu_B - (C+mP)/(r+m) \\ P - (C+mP)/(r+m) \end{bmatrix} \end{aligned} \tag{30}$$

The rest of the formulas at the issuance point are:

$$E(\nu) = \nu + DB(\nu) - D(\nu) \tag{31}$$

$$W(\nu) = DB(\nu_I)(\frac{\nu}{\nu_I})^{\beta_1}, \quad \nu \le \nu_I \tag{32}$$

$$\nu_B : \frac{\partial E}{\partial \nu_B} = 0 \tag{33}$$

$$\phi : \frac{\partial DB}{\partial \phi} = 0, \quad \phi > 1 \tag{34}$$

$$P^* : \frac{\partial DB}{\partial P} = 0, \quad P \ge 0, C = rP \tag{35}$$

$$\nu_I : \frac{\partial DB(\nu_0)}{\partial \nu_0}|_{\nu_0=\nu_I} = \beta_1 \frac{DB(\nu_0)}{\nu_0}|_{\nu_0=\nu_I} \tag{36}$$

Finding the optimal barrier ($\nu_B : \partial E/\partial \nu_B = 0$) leads to the calculation of the optimal leverage by choosing the optimal face value ($P^* : \partial DB/\partial P|_{P=P^*} = 0$)[29] and the optimal

[29] The optimal leverage is calculated *post issuance*. The decision for optimal leverage does not change

rebalancing point by choosing the optimal upper bound ($\phi : \partial DB/\partial\phi = 0$). Optimal default barrier is the function of optimal leverage and rebalancing point. Both optimal leverage and the rebalancing point are functions of the issuance threshold ($\nu_I : \partial DB/\partial\nu_0 = \beta_1 DB(\nu_0)/\nu_0$).

D More simulation details for Case I

The unlevered asset value is simulated as:

$$\nu_t = \nu_{t-\Delta t} exp((\mu - \delta - \frac{\sigma^2}{2})\Delta t + \sqrt{\Delta t}(\sigma_O z_t^O + \beta\sigma_S z_t^S)) \tag{37}$$

where z^O and z^S are standard normal shocks to the firm and the economy and Δt is one quarter. Although the model is discretized, the process is time continuous. For simplicity, I assume that once the firm hits a decision threshold in between the quarters, e.g. a rebalancing point, the process stops and remains constant until the decision is executed at the quarter end. The assumption substantially reduces the amount of memory and processing time required by the simulation for the traditional and real-option models.

E Hypothesis testing on simulated data

I validate the hypotheses about the effect of volatility (H1), bankruptcy costs (H2), payout rate (H4) and size (H5) on the ZL choice in the simulation data. Since tax rate is constant in the simulated data for brevity, I only do not test the hypothesis related to tax savings (H3). In each simulated economy, I run PROBIT regression on the the firms' choice of ZL policy to check the effect of the factors. There are 201 quarters for 500 firms in each economy. The regression looks at the propensity of a firm to follow ZL depending on the

before or after the issuance and the managers commit to their decision. Although the result is intuitive, see Proposition 1 in Online Appendix I.6 for a formal proof).

independent factors. The regression has the following form:

$$\begin{aligned}&\Pr(\mathrm{ZL}) = N\big[d_0 + d_1\log(\sigma) + d_2\text{PBC rate} + d_3\text{Asset payout rate} + d_4\text{size} + d\Theta\big]\\&\Theta = \{\text{other model parameters such as debt retirement rate, } m\text{, etc.}\}\end{aligned} \tag{38}$$

where d represents the regression coefficients. Using the volatility without log-transformation does not change the inference results. However, I use log-transformation on the volatility because it guarantees positive estimation for the volatility boundary as later explained for Equation 8. After running the choice regression in each economy, I report the distribution properties of the coefficients in Table 12. In line with the four hypotheses presented in Section 2.1.2, high volatility, small size, high bankruptcy cost rate, and low asset payout rate increase the propensity of the firm to follow ZL policy in the simulations. In addition, the results show the low power of the test on whether the coefficients are different from zero for some variables such as the debt costs. For example, while in Figure 4a the waiting ratio is longer for the firms with higher bankruptcy cost, PROBIT regression does not reject the null of the coefficient being different from zero at 5%-95% confidence interval. It is possibly due to estimating a linear equation for a non-linear relation between bankruptcy cost and the ZL policy.

[**Place Table 12 about here**]

F The bijection between the volatility boundary and the asset threshold

Let's assume the asset threshold, ν_I, has a unique solution in Equation 4, and the threshold is increasing in the volatility. The assumption is valid, if the net debt benefit function, DB, is always continuously and monotonically decreasing in asset volatility and increasing in the unlevered asset. Then, $\nu_I(.)$ has an inverse function. It is trivial to mathematically show the bijection that exists between the unlevered threshold, $\nu_I(.)$, and the volatility boundary,

Table 12- Descriptive statistics on PROBIT regression coefficients for the factors affecting the ZL choice in the simulations: Data is from the simulated firm-quarters in Section 2.1.1. There are 1000 economies with 201 quarters simulated for 500 firms. In each economy, I run the PROBIT which looks at the factors leading to the choice of having ZL:Pr(ZL) $= N\left[d_0 + d_1\log(\sigma) + d_2\text{PBC rate} + d_3\text{Asset payout rate} + d_4\text{size} + d\Theta\right]$ where $\Theta = \{$other model parameters such as debt retirement rate, m, etc.$\}$. ZL (Zero-Leverage) is equal to 1 (yes) in a firm-quarter with zero leverage and 0 (no) otherwise. On average, d_1 has a positive sign as in H1, d_2 has a positive sign as in H2, d_3 has a negative sign as in H4, and d_4 has a negative sign as in H5. I do not test H3 because it is constant in the simulations.

parameter	Median	Mean	Std Dev	5th Pcl	95th Pcl	p-value
Intercept	1.9	1.8	2.8	-2.3	5.7	21.0%
Log(asset volatility)	5.4	6.7	5.9	2.4	15.6	0.0%
Bankruptcy cost rate	1.3	1.8	4.5	-2.8	6.9	28.3%
Asset payout rate	-22.2	-26.9	33.2	-64.7	5.4	8.4%
Size	-0.019	-0.026	0.023	-0.073	-0.002	0.0%
Debt retirement rate, m	8.4	9.5	14.1	-6.3	30.1	15.8%
Rollover cost, k_2	-66.5	-64.5	179.4	-314.2	149.5	28.7%
Issuance cost rate, k_1	69.3	92.3	223.2	-196.2	414.4	31.0%
Fixed issuance cost, κ	37.0	49.6	60.7	-7.6	153.1	8.7%

$\sigma_I(.)$:

$$ZL = \begin{cases} 1 & \text{if} \quad \nu_0 \leq \nu_I(\sigma, \Theta) \Leftrightarrow \sigma \geq \sigma_I(\nu_0, \Theta) \Leftrightarrow \Psi(\nu_0, \sigma, \Theta) \geq 0, \\ 0 & \text{if} \quad Otherwise \end{cases} \tag{39}$$

$$\Theta = \{\text{other model parameters such as PBC rate } (\alpha), \text{ etc.}\}$$

Intuitively, consider all the parameters are constant except the unlevered value and the volatility. For a given volatility, a firm will decide to follow ZL policy, if its current unlevered value is below the threshold. The threshold divides the unlevered value cross-section into two groups of issuing and non-issuing firms. For a given unlevered value, the threshold turns into a volatility boundary above which waiting is optimal. The boundary divides the volatility cross-section into two groups of issuing and non-issuing firms. The volatility boundary in the volatility cross-section is a divider equivalent to the unlevered threshold in the size cross-section. One is calculable, given the other, and there is bijection between them. Figure 9 shows the bijection. It is analogous to American call options' exercise where the underlying stocks' prices are the same and the investors decide to exercise the options depending on the underlying stocks' volatility with respect to the volatility boundary.

[Place Figure 9 about here]

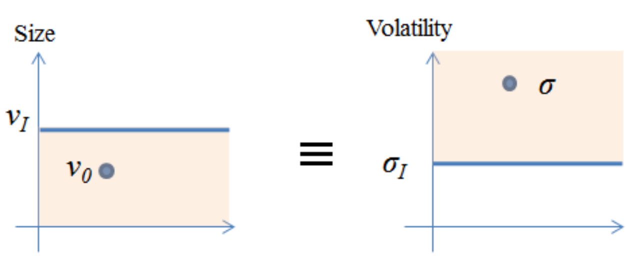


Figure 9. The bijection between the asset threshold (ν_I) and the volatility boundary (σ_I)

Because of the bijection between the volatility boundary and the unlevered threshold, wherever this article mentions a hypothesis, it also can express parallel expressions in terms of the volatility boundary. For example, in case of H2, the parallel expressions are: the volatility boundary (unlevered threshold) above (below) which managers prefer to wait for debt issuance decreases (rises) when the bankruptcy-cost increases, *ceteris paribus*. There are similar other bijections with respect to the other parameters, e.g. the tax rate. However, this article only focuses on volatility-size bijection.

G Case II: The real option to have debt in static trade-off and capital structure

The debt structure is very similar to Leland (1994) which is a static model: there are no rollover risk, no maturity, no convertible debt, no debt call back, no liquidity, no rebalancing, and no conflicts of interests. Hence, the model in Case I nests this model. Consider an all-equity firm holding the option to issue a consol bond only once with continuous coupon payments, C. Here, the only debt cost is the bankruptcy cost. The cost has two components: a cost proportional to the unlevered assets' value at default, $\alpha\nu_B$, and a fixed cost, K.[30] The managers make three decisions: a) optimal timing of debt issuance with ν_I b) optimal leverage at issuance through optimal coupon, C^*, c) optimal default barrier, ν_B. The managers trade-off between tax savings and bankruptcy costs to find the optimal leverage. The optimal default barrier is activated after having leverage. The decision variable for the default barrier is the unlevered asset value below which the shareholders will stop serving the coupons.

Debt, tax saving, bankruptcy cost and equity values depend on the unlevered asset value. They follow the no-arbitrage PDE and the Dirichlet conditions. See Online Appendix I.5 for derivation details. After the derivation of the values using PDE, the tax savings and bankruptcy costs are:

$$DC(\nu) = BC(\nu) = (\alpha\nu_B + K)(\frac{\nu}{\nu_B})^{-\beta_2} \tag{40}$$

$$TS(\nu) = \frac{\tau C}{r}(1 - (\frac{\nu}{\nu_B})^{-\beta_2}) \tag{41}$$

[30]The fixed cost assumption is similar to the earlier studies and creates non-convex debt costs. For a similar assumption, see Mao and Tserlukevich (2014), Anderson and Sundaresan (2000) and Dotan and Ravid (1985).

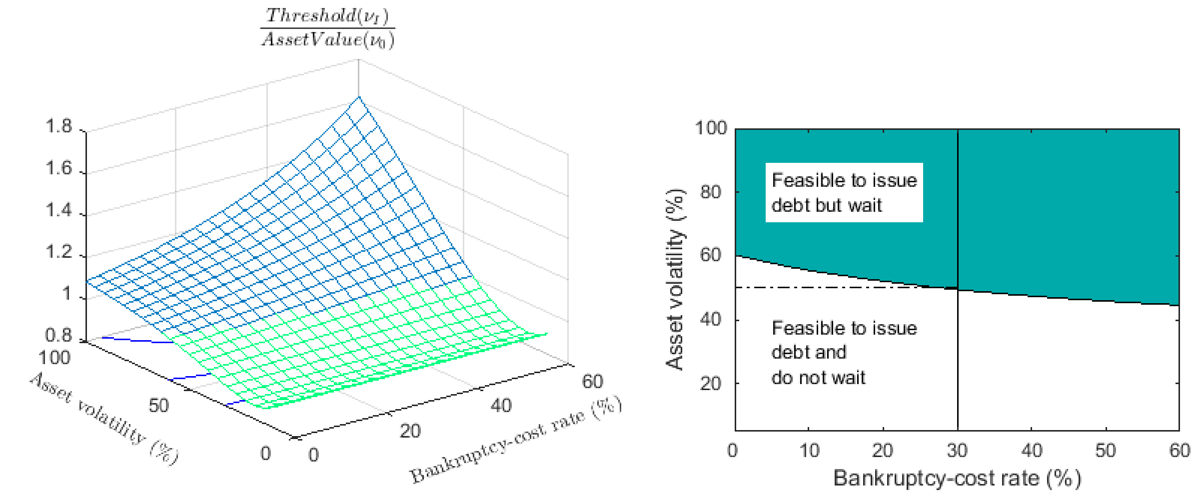
where TS is the present value of tax savings, τ is the tax rate, and BC is the present value of bankruptcy cost at the issuance point. All the parameters such as the optimal default boundary, ν_B, are available in Online Appendix I.5. Managers choose the default barrier to maximize equity value and choose the optimal leverage to maximize the total debt benefits of the firm at issuance. Again, the model is a classic trade-off model up to finding the optimal leverage. After finding the optimal leverage, the model numerically calculates the optimal threshold to have debt, ν_I, using Equation 4 where $DB(\nu) = TS(\nu) - DC(\nu)$ from Equations 40 and 41. The optimal default and leverage depend on when the firm decides to have debt. After finding the optimal default barrier, the model calculates the optimal leverage and, then, the optimal issuance threshold. In the calibrations and comparisons of Case II, the parameters are close to the reported values by the earlier case. The total bankruptcy cost rate is the sum of the fixed and proportional bankruptcy costs divided by the default barrier. The cost rate ranges from 30% to 45%, depending on the optimal default barrier.

The next figures are the results of the calibrations. They support the five hypotheses from Case I calibrations. Figure 10 is a two-dimensional analysis and shows the increasing trend of waiting with respect to the asset volatility and bankruptcy cost. Figure 11 shows the negative relation between the waiting ratio, ν_I/ν_0, and the independent variables, i.e. tax rate, payout rate and size. At 30% PBC rate, the overall bankruptcy cost rate is about 40% and the model with the real option implies ZL for volatility higher than 50%. At 50% asset volatility, the traditional model without the real option only produces ZL policy with 100% default costs, and low tax savings, 5%, which are away from the observed empirical values. This result is similar to the earlier finding in Case I. Hence, the real option has first-order effect similar to bankruptcy costs on generating ZL policy.

[Place Figure 10 about here]

[Place Figure 11 about here]

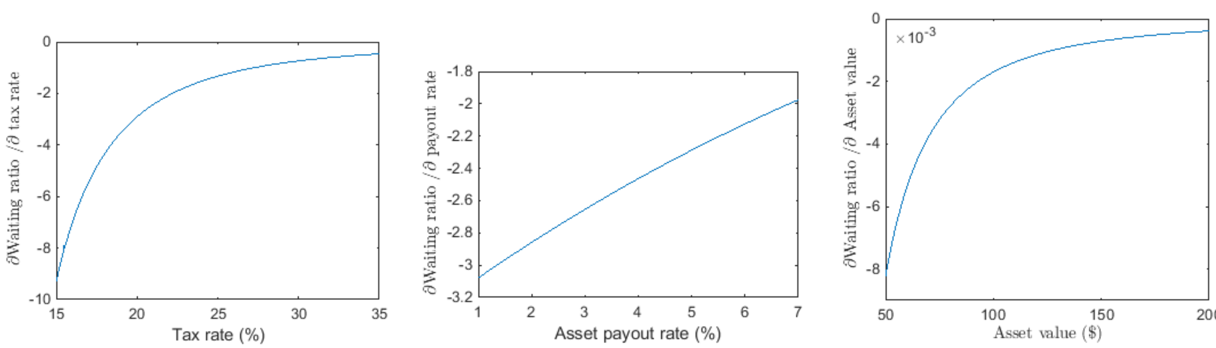
The idea of treating debt issuance as a real option requires only concave debt cost structure and works well with other costs in the literature. In order to support this claim, I



(a) The waiting ratio (ν_I/ν_0) as a function of volatility and the bankruptcy cost.

(b) ZL policy in the cross-section for volatility and the bankruptcy cost

Figure 10. ZL policy in the two-dimensional cross-section for volatility and bankruptcy cost: The figures are comparable to Figure 4. The X-axis shows the proportional bankruptcy cost (PBC) rate. The Y-axis shows the firm's asset volatility. The Z-axis shows the waiting ratio as the ratio of the optimal recapitalizing threshold to the firm's value. A ratio above 1 implies that the firm waits to issue debt as seen in the contour plot. Figure 10b is generated by a look to Figure 10a from the top. In Figure 10b, dependent decision variable is to follow ZL policy. Firms in the dark area follow ZL policy even when having debt has a positive net value (feasible) and traditional model predicts that these firms should have leverage. Firms in the white area simply issue debt. The risk-free rate (r) is 5%, the payout rate (δ) is 3%, the unlevered value (ν_0) is \$100, the fixed bankruptcy cost (K) is \$2, and the tax rate (τ) is 25%. At 30% PBC rate and volatility of 50%,the overall bankruptcy cost rate is about 40% which is comparable to a similar number in Figure 4.



(a) **(b)** **(c)**

Figure 11. The effect of the independent variables on the waiting ratio in Case II: The figures are comparable to Figure 5. The X-axis shows the independent variable. The Y-axis shows the slope in the waiting ratios, $\frac{\partial(\nu_I/\nu_0)}{\partial x}$, where x is the independent variable. All the slopes are negative which support the hypotheses, H3, H4 and H5. In all the figures the volatility is 0.6. All the other calibration parameters match with Figure 10.

slightly change the model. All the assumptions are identical. Except, the fixed bankruptcy cost is dropped and a fixed issuance (transaction, or flotation) cost, κ, is added to the model. To have a very simple structure for the concave cost, the firm incurs a one-time fixed issuance cost.

Details of the derivations are in Online Appendix I.5. Tax savings are the same as Equation 41. Debt costs are slightly different:

$$DC(\nu) = BC(\nu; K = 0) - \kappa, \qquad DB(\nu) = TS(\nu) - DC(\nu) \tag{42}$$

The model uses Equation 4 to calculate the issuance threshold. The total debt costs have two elements in the formula, the fixed issuance cost and proportional bankruptcy cost.

As before, the new model with issuance cost implies the same hypotheses because its calibrations create similar results. For example, the model calibrations on the volatility and bankruptcy cost cross-section result in Figure 12 which is similar to Figure 10b and yields H1 and H2. The other results and figures are also similar and are omitted for brevity. For the calibration, the constant flotation cost is $1, smaller than 2% of the debt's market value. At 30% bankruptcy cost rate, the traditional model without the real option requires an issuance cost which is 7.3 times higher (14.6% flotation cost) to create ZL policy. Hence, the model with the real option produces ZL policy under more reasonable costs compared to the traditional model.

[**Place Figure 12 about here**]

Here, requiring only concave debt costs is an important result. The traditional trade-off models ignore the decomposition of the costs for answering the question "To be or not to be leveraged?". They only consider the positivity of the net debt benefits. As long as the net debt benefits are positive, the traditional models still suggest immediate debt issuance, regardless of the fixed debt costs. However, this article argues that the debt cost structure combined with the real option matters in deciding to have debt.

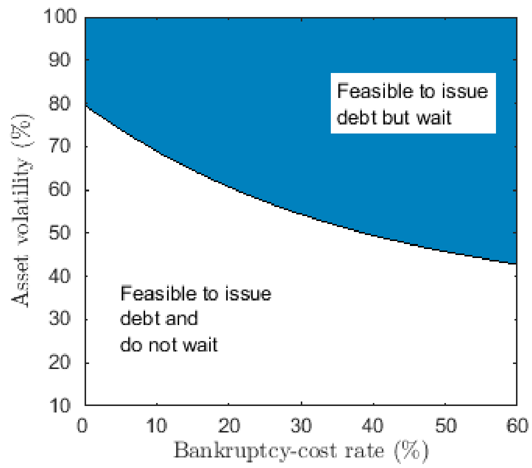


Figure 12. ZL policy in the two-dimensional cross-section for volatility and bankruptcy cost in Case II with issuance cost: The figure is comparable to Figure 4b and Figure 10b. The X-axis shows the PBC rate. The Y-axis shows the firm's asset volatility. Dependent decision variable is to follow ZL policy. Firms in the dark area follow ZL policy even when having debt has a positive net value (feasible) and traditional model predicts that these firms should have leverage. Firms in the white area simply issue debt. The risk-free rate is 5%, the asset payout rate is 3%, the tax rate is 25%, the constant issuance (flotation) cost, κ, is \$1 (smaller than 2 %) and the unlevered value is \$100.

H Controlling for the governance

Inferences about the hypotheses do not change as the article controls for CEO ownership and governance quality of the firms. Table 13 shows all the results. Governance variables substantially reduces the size of the dataset due to limited data availability. The sample with both governance and CEO ownership variables has less than 15% of the firm-quarters in the original sample, which also reduces the test power. For CEO ownership, I use Compustat's executive compensation data which includes CEO's shareholdings excluding options (SHROWN_EXCL_OPTS). CEO ownership is the CEO's shares divided by the total shares. Not all of the firms have the data available and the sample almost halves. Gompers, Ishii, and Metrick (2003) provide governance index for some firms in their online database and this article uses the index as an indicator of the governance quality. A high value for the index implies low governance quality. The index data is available bi-annually. For missing data, I linearly interpolate the index values for the firms. Then, I rank the sample based on high and low index values and define a dummy of 1 for the firms with high values. The rank dummy reduces the possibility of having the interpolation influence the regression results.

The governance results are similar to the earlier empirical studies. Along with Strebulaev and Yang (2013), the regressions show that high CEO ownership can increase the propensity of a firm to follow ZL policy. The high ownership increases the cost of default for the manager-owner because the manager has low-diversified personal portfolio and high stakes in the firm. Low governance quality also has a similar effect. Moreover, controlling for ownership and the quality of governance does not substantially change the inferences related to the 5 hypotheses in the article, especially for the asset volatility which is related to the real-option idea.

[**Place Table 13 about here**]

Table 13- Regression results on ZL policy with controlling for governance variables The regressions estimate Pr(ZL) $= N[a_0 + a_1\log(\text{volatility}) + a_{2a}\text{Tangibility} + a_{2b}\text{BM ratio} + b_1\text{Profitability} + b_2\text{Tax Ratio} + a_3\text{Profitability} \times \text{Tax Ratio} + a_{4a}\text{Div. dummy} + a_{4b}\text{Payout Rate} + a_5\log(\text{size}) + b_3 \text{ cash} + b_4 \text{ CEO ownership} + b_5 \text{ Governance quality} + b_6 \text{ Governance*Ownership} + \textit{dummies}]$. $N[.]$ is standard cumulative normal distribution. Dummies represent years, industries, and fiscal quarters. Expected signs are: a_1 to be positive and a_{2a}, a_{2b}, a_3, a_{4a}, a_{4b}, and a_5 to be negative. Four hypotheses are supported by the results: High volatility (a_1), high debt costs(a_{2a}, a_{2b}), low tax payments (a_3), and small size (a_5) increase the propensity to remain ZL. Only a_{4a} has a different sign and a_{4b} is not significant which are related to H4. The p-values test the null hypothesis that the coefficient is zero. CEO ownership is the CEO's shares divided by the total shares. GIM rank is a dummy with value of 1 for the firms with relative high index value and low governance quality.

Parameter	Estimated Coefficients for each statistical regression			
	Controlling for CEO ownership		Controlling for CEO ownership and governance quality	
Model	(1)	(2)	(3)	(4)
Obs	25,580	25,580	6,945	6,945
Intercept	0.71	1.09	0.46	0.35
p-value	(0.00)	(0.00)	(0.41)	(0.57)
log volatility	0.40	0.70	0.64	0.65
p-value	(0.00)	(0.00)	(0.02)	(0.05)
Tangibility	-0.34	0.12	-0.54	-0.61
p-value	(0.11)	(0.75)	(0.54)	(0.51)
B/M ratio	-0.41	-0.84	-1.30	-1.28
p-value	(0.00)	(0.00)	(0.02)	(0.05)
Profitability	1.62	0.52	0.62	0.55
p-value	(0.03)	(0.71)	(0.85)	(0.87)
Tax rate	0.13	0.09	0.12	0.11
p-value	(0.00)	(0.00)	(0.16)	(0.26)
Profit*Tax	-2.14	-0.29	-5.79	-5.08
p-value	(0.13)	(0.87)	(0.22)	(0.33)
Div. Pay. Dummy	0.27	0.37	0.46	0.47
p-value	(0.00)	(0.00)	(0.01)	(0.04)
Payout rate	-0.07	0.01	0.11	0.09
p-value	(0.43)	(0.90)	(0.61)	(0.71)
Log(Size)	-0.12	-0.14	-0.08	-0.07
p-value	(0.00)	(0.00)	(0.17)	(0.26)
Cash	1.66	1.92	1.75	1.72
p-value	(0.00)	(0.00)	(0.01)	(0.04)
Ceo ownership	-	0.92	-	1.46
p-value	-	(0.02)	-	(0.16)
GIM rank	-	-	-	0.08
p-value	-	-	-	(0.75)
GIM rank* CEO ownership	-	-	-	-1.33
P-value	-	-	-	(0.31)
Time, Industry, Fiscal Quarter dummies	yes	yes	yes	yes
Control clustered time and industry errors	yes	yes	yes	yes
Pseudo R^2	33.9%	34.1%	40.4%	40.8%

I Online Appendices

I.1 Simulation robustness check for the low-volatility sample

This appendix shows that the simulation results in generating ZL observations for the model with the real option is robust to volatility distribution. Table 14 presents the results that are comparable to Table 3. While average volatility of %25 used by earlier studies is based on all rated firms, the model is able to produce %8 ZL observations in this setting. However, the simulation for the traditional model shows no ZL observations.

[**Place Table 14 about here**]

I.2 Robustness check in the sample of almost zero-leveraged firms

I show that the empirical inferences about the hypotheses in the choice regression is also robust to the definition of the zero leverage. Almost zero-leveraged firms (AZL) are the firms with extremely low leverage close to zero. Following Strebulaev and Yang (2013), I define AZL firms as the firms with the leverage below 5% at least during one quarter between 1996 and 2012. The other filters and the data gathering procedure are similar to ZL firms. For example, I drop financial and utility firms and the firm-quarters with book assets smaller than $10 Millions. Then, I run the PROBIT regression on the sample to test the 5 hypotheses. There is stronger support for the hypotheses, especially for coefficients of tangibility and taxes, with higher pseudo R-squared (compare Table 15 with Table 8).

[**Place Table 15 about here**]

I.3 Robustness check in the pooled samples of ZL and levered firms

This section shows that the results of PROBIT regressions are robust when both samples of ZL and levered firm-quarters are pooled together. I combine the samples and then run

Table 14- Simulation results for the dynamic capital structure model: The table is comparable to Table 3. The table reports the distribution of the market leverage, quasi-mark leverage, zero leverage firms and average net debt benefits from the simulation. The table compares the traditional dynamic trade-off model without the real option and the model with the real option of optimal timing to have debt, as described in Case I. All parameters are set to values in Table 2, except the average asset volatility is lower at 27% with %14 standard deviation. The statistics are based on simulating observations of 1000 economies with 500 firms for 201 quarters (50 years). The first section reports the statistics after dropping the first 25 years of observations from the simulation. The mean, standard deviation and the percentiles for the leverages are reported for all the observations. Market leverage is the ratio of the market value of debt to the market value of the firm. Quasi-market leverage is the ratio of book value of debt to the sum of equity's market value and the book value of debt. Average number of ZL firms is reported after averaging the ZL observations in each quarter within each economy and then calculating the mean for all the economies. Net debt benefits are the tax savings less the debt issuance and default costs. Average benefits are reported after averaging them for each firm in each economy and then calculating the mean for all the economies. The second section includes all the observations simulated. The last section only reports the statistics of the observations across the firms in the initial quarter (time 0) for all the economies.

			Percentiles							
	Mean	Std	5	10	15	20	50	85	90	95
100 Quarters in the last 25 years of the simulated data										
Market leverage (ML)										
The real option model	0.24	0.15	0	0.06	0.10	0.12	0.22	0.37	0.42	0.51
The real option model without ZL	0.26	0.14	0.08	0.11	0.13	0.15	0.23	0.38	0.43	0.52
Traditional model	0.25	0.15	0.07	0.10	0.12	0.13	0.23	0.38	0.43	0.52
Quasi-Market leverage (QML)										
The real option model	0.24	0.16	0	0.06	0.10	0.12	0.22	0.38	0.43	0.53
The real option model without ZL	0.26	0.15	0.09	0.11	0.13	0.15	0.24	0.39	0.44	0.55
Traditional model	0.26	0.16	0.07	0.10	0.12	0.14	0.23	0.40	0.45	0.57
Average number of ZL firms in each quarter	Mean	Std	Average net debt benefits					Mean		Std
Traditional model	0	0	Traditional model					0.06		0.01
The real option model	0.08	0.05	The real option model					0.06		0.01
Complete simulated data including the initial values at time 0										
Market leverage (ML)										
The real option model	0.24	0.15	0	0.07	0.10	0.12	0.22	0.37	0.41	0.49
The real option model without ZL	0.25	0.14	0.08	0.10	0.12	0.14	0.23	0.37	0.42	0.50
Traditional model	0.25	0.14	0.06	0.09	0.11	0.13	0.22	0.37	0.42	0.50
Initial values at time 0										
Market leverage (ML)										
The real option model	0.23	0.11	0.05	0.09	0.12	0.14	0.22	0.34	0.38	0.44
The real option model without ZL	0.24	0.11	0.08	0.11	0.13	0.15	0.23	0.35	0.38	0.44
Traditional model	0.23	0.11	0.05	0.09	0.12	0.14	0.22	0.34	0.38	0.44

Table 15- PROBIT regression results : It estimates $\Pr(\text{ZL}) \propto a_0 + a_1\log(\text{volatility}) + a_{2a}\text{Tangibility} + a_{2b}\text{BM ratio} + b_1\text{Profitability} + b_2\text{Tax Ratio} + a_3\text{Profitability}\times\text{Tax Ratio} + a_{4a}\text{Div. dummy} + a_{4b}\text{Payout Rate} + a_5\log(\text{size}) + b_3$ cash + *dummies*. Dummies represent years, industries, and fiscal quarters. The sample includes all the firms with almost zero leverage (AZL). AZL firms are the firms that at least once have leverage below 5% between 1996 and 2015. Expected signs are: a_1 to be positive and a_{2a}, a_{2b}, a_3, a_{4a}, a_{4b}, and a_5 to be negative. Most of the hypotheses are supported by the results: High volatility (a_1), high debt costs(a_{2a}, a_{2b}), low tax payments (a_3), and small size (a_5) increase the propensity to remain AZL. Only a_{4a} related to H4 has a different sign and and a_{4b} is not significant. The p-values test the null hypothesis that the coefficient is zero.

Parameter	Estimated Coefficients for each statistical regression		
Model	(1)	(2)	(3)
Intercept	-0.73	-0.73	0.34
p-value	(0.00)	(0.00)	(0.00)
log volatility	0.36	0.36	0.25
p-value	(0.00)	(0.00)	(0.00)
Tangibility	-0.68	-0.68	-0.55
p-value	(0.00)	(0.00)	(0.00)
B/M ratio	-0.47	-0.47	-0.37
p-value	(0.00)	(0.00)	(0.00)
Profitability	2.70	2.70	3.83
p-value	(0.00)	(0.00)	(0.00)
Tax rate	0.21	0.21	0.24
p-value	(0.00)	(0.00)	(0.00)
Profit*Tax	-4.57	-4.57	-6.88
p-value	(0.00)	(0.01)	(0.00)
Div. Pay. Dummy	0.27	0.27	0.31
p-value	(0.00)	(0.00)	(0.00)
Payout rate	0.004	0.004	0.001
p-value	(0.59)	(0.78)	(0.81)
Log(Size)	-0.18	-0.18	-0.18
p-value	(0.00)	(0.00)	(0.00)
Cash	1.84	1.84	1.63
p-value	(0.00)	(0.00)	(0.00)
Time, Industry, Fiscal Quarter dummies	yes	yes	no
Control clustered time and industry errors	no	yes	no
Pseudo R^2	36%	36%	24%

the PROBIT regression on the whole sample. The results support most of the hypotheses on the choice of the firms to follow ZL policy with higher R-squared (compare Table 16 with Table 8).

[Place Table 16 about here]

I.4 Robustness check in using a different method to calculate volatility

This section shows that the results of PROBIT regressions are robust when asset volatility is measured by 91-day historical volatility from stock prices for ZL firm-quarters. Volatility is delevered for non-ZL firm-quarters. The results support all the five hypotheses on the choice of the firms to follow ZL policy (compare Table 17 with Table 8).

[Place Table 17 about here]

I.5 Derivation of the formulas in Case II

Debt, equity, tax savings, and debt costs are all claims defined on the unlevered asset value and follow the PDE in Equation 18. For debt, it follows:

$$D(\nu_B) = ((1-\alpha)\nu_B - K)^+, \qquad D(\nu \to \infty) = \frac{C}{r} \tag{43}$$

$$D(\nu) = \frac{C}{r} + \left[\left((1-\alpha)\nu_B - K\right)^+ - \frac{C}{r}\right](\frac{\nu}{\nu_B})^{-\beta_2}, \tag{44}$$

The last term in the debt formula is the discounted RA default probability. $(.)^+$ is due to limited liability because the value of the firm at default cannot fall below zero. The calibrations set the PBC rate and the fixed cost small enough to meet limited liability. Hence, I drop the positive part of the payoff, $(.)^+$, in the debt formula.[31] The Dirichlet

[31] The condition is $K \leq (1-\alpha)\frac{C-\tau C}{r}(\frac{\beta_2}{1+\beta_2})$ to ensure $(1-\alpha)\nu_B - K \geq 0$.

Table 16- PROBIT regression results : It estimates Pr(ZL) $\propto a_0 + a_1\log(\text{volatility}) + a_{2a}\text{Tangibility} + a_{2b}\text{BM ratio} + b_1\text{Profitability} + b_2\text{Tax Ratio} + a_3\text{Profitability}\times\text{Tax Ratio} + a_{4a}\text{Div. dummy} + a_{4b}\text{Payout Rate} + a_5\log(\text{size}) + b_3\ \text{cash} + \textit{dummies}$. Dummies represent years, industries, and fiscal quarters. The sample pools all the ZL firms and all-time levered firms together. Expected signs are: a_1 to be positive and a_{2a}, a_{2b}, a_3, a_{4a}, a_{4b}, and a_5 to be negative. Most of the hypotheses are supported by the results: High volatility (a_1), high debt costs(a_{2a}, a_{2b}), low tax payments (a_3), and small size (a_5) increase the propensity to remain ZL. Only a_{4a} related to H4 has a different sign and a_{4b} is not significant. The p-values test the null hypothesis that the coefficient is zero.

Parameter	Estimated Coefficients for each statistical regression		
Model	(1)	(2)	(3)
Intercept	-1.21	-1.21	0.50
p-value	(0.00)	(0.00)	(0.00)
log volatility	0.51	0.51	0.44
p-value	(0.00)	(0.00)	(0.00)
Tangibility	-0.82	-0.82	-0.80
p-value	(0.00)	(0.00)	(0.00)
B/M ratio	-0.58	-0.58	-0.54
p-value	(0.00)	(0.00)	(0.00)
Profitability	2.78	2.78	3.94
p-value	(0.00)	(0.00)	(0.00)
Tax rate	0.02	0.02	0.02
p-value	(0.02)	(0.01)	(0.03)
Profit*Tax	-0.69	-0.69	-0.87
p-value	(0.04)	(0.01)	(0.03)
Div. Pay. Dummy	0.31	0.31	0.35
p-value	(0.00)	(0.00)	(0.00)
Payout rate	0.002	0.002	0.007
p-value	(0.69)	(0.82)	(0.31)
Log(Size)	-0.19	-0.19	-0.18
p-value	(0.00)	(0.00)	(0.00)
Cash	1.97	1.97	1.75
p-value	(0.00)	(0.00)	(0.00)
Time, Industry, Fiscal Quarter dummies	yes	yes	no
Control clustered time and industry errors	no	yes	no
Pseudo R^2	46%	46%	35%

Table 17- PROBIT regression results : It estimates $\Pr(\text{ZL}) \propto a_0 + a_1 \log(\text{volatility}) + a_{2a}\text{Tangibility} + a_{2b}\text{BM ratio} + b_1\text{Profitability} + b_2\text{Tax Ratio} + a_3\text{Profitability} \times \text{Tax Ratio} + a_{4a}\text{Div. dummy} + a_{4b}\text{Payout Rate} + a_5\log(\text{size}) + b_3 \text{ cash} + \textit{dummies}$. Dummies represent years, industries, and fiscal quarters. In the sample, asset volatility is delevered volatility from historical stock prices during the past 91 days. Expected signs are: a_1 to be positive and a_{2a}, a_{2b}, a_3, a_{4a}, a_{4b}, and a_5 to be negative. All the hypotheses are supported by the results: High volatility (a_1), high debt costs(a_{2a}, a_{2b}), low tax payments (a_3), low payout rate (a_{4b}) and small size (a_5) increase the propensity to remain ZL. Only a_{4a} related to H4 has a different sign. The p-values test the null hypothesis that the coefficient is zero.

Parameter	Estimated Coefficients for each statistical regression	
Model	(1)	(2)
Intercept	0.60	0.60
p-value	(0.00)	(0.01)
log historical volatility	0.24	0.24
p-value	(0.00)	(0.00)
Tangibility	-0.35	-0.35
p-value	(0.00)	(0.10)
B/M ratio	-0.44	-0.44
p-value	(0.00)	(0.00)
Profitability	1.38	1.38
p-value	(0.00)	(0.08)
Tax rate	0.12	0.12
p-value	(0.00)	(0.00)
Profit*Tax	-2.18	-2.18
p-value	(0.05)	(0.10)
Div. Pay. Dummy	0.25	0.25
p-value	(0.00)	(0.00)
Payout rate	-0.09	-0.09
p-value	(0.09)	(0.34)
Log(Size)	-0.16	-0.16
p-value	(0.00)	(0.00)
Cash	1.72	1.72
p-value	(0.00)	(0.00)
Time, Industry, Fiscal Quarter dummies	yes	yes
Control clustered time and industry errors	no	yes
Pseudo R^2	26%	26%

conditions and solutions for tax savings, TS, and bankruptcy costs, BC, are:

$$TS(\nu_B) = 0, \qquad TS(\nu \to \infty) = \frac{\tau C}{r} \tag{45}$$

$$BC(\nu_B) = \alpha\nu_B + K, \qquad BC(\nu \to \infty) = 0 \tag{46}$$

$$TS(\nu) = \frac{\tau C}{r}(1 - (\frac{\nu}{\nu_B})^{-\beta_2}) \tag{47}$$

$$DC(\nu) = BC(\nu) = (\alpha\nu_B + K)(\frac{\nu}{\nu_B})^{-\beta_2} \tag{48}$$

where $DB(\nu) = TS(\nu) - DC(\nu)$. The equity holders are the residual claim holder of the levered firm after deducting debt value:

$$E(\nu) = \nu + TS(\nu) - BC(\nu) - D(\nu) \tag{49}$$

E is the equity value. Mathematically, solving the first order condition (FOC) for the optimal default barrier yields the formula similar to Leland (1994).:

$$\frac{\partial E}{\partial \nu_B} = 0 \Rightarrow \nu_B = \frac{C - \tau C}{r}(\frac{\beta_2}{1 + \beta_2}) \tag{50}$$

Finding the optimal barrier leads to the calculation of the optimal leverage by choosing the optimal coupon ($C^* : \partial(DB)/\partial C|_{C=C^*} = 0$). Both optimal leverage and the default barrier are functions of the issuance threshold.

The Dirichlet conditions and solution for the real option are:

$$\begin{gathered} W(0) = DB(0) = 0, \quad \nu_I \leq \nu : W(\nu) = DB(\nu) = TS(\nu) - DC(\nu), \\ \nu \leq \nu_I : W(\nu) = DB(\nu_I)(\frac{\nu}{\nu_I})^{\beta_1}, \end{gathered} \tag{51}$$

I.5.1 Case II with issuance cost

In the case with issuance costs, all the formulas are similar to Case II (see Equations 40, 41, and 44) where the fixed default cost is set equal to zero ($K = 0$). Only the equity formula has an extra term for the fixed issuance cost:

$$E(\nu) = \nu + TS(\nu) - BC(\nu) - D(\nu) - \kappa \tag{52}$$

The optimal default barrier and leverage satisfy:

$$\nu_B : \frac{\partial E}{\partial \nu_B} = 0, \quad C^* : \frac{\partial (DB)}{\partial C}|_{C=C^*} = 0 \tag{53}$$

The optimal default barrier is the same as Equation 50 after solving for ν_B. The optimal leverage is slightly different because the net debt benefit function is slightly different from the case without issuance costs. Debt benefit with issuance cost is:

$$DC = BC(\nu) - \kappa, \qquad DB(\nu) = TS(\nu) - [BC(\nu) - \kappa] \tag{54}$$

I.6 Proposition 1 and the proof

For the optimal issuance timing, this appendix shows that the optimal leverage (or face value of debt, P) *ex ante* is the same as the optimal leverage (or face value of debt, P) *ex post*. Although the equivalence between the optimal leverages *ex ante* and *ex post* seems intuitive, this appendix formally proves it.[32] In the model and prior to issuing debt, the issuance threshold depends on the optimal face value of debt. The optimal face value of debt also depends on the decision about the issuance threshold because higher face value implies a higher threshold. Managers decide about both parameters before the issuance. After the issuance, however, managers decide only about the optimal face value. The proposition here

[32] By changing the optimal face value, P, into coupon rate, C, the same proof and proposition apply to Case II.

rules out any possibility that managers would optimally choose a different post-issuance face value of debt.

PROPOSITION 1. *The optimal face value of debt (or leverage) prior to debt issuance is equal to the optimal face value of debt after the decision to issue debt.*

Proof. First, let's check the FOC for optimal face value after issuance. *Ex post*, the option is already exercised and ν_I is constant. The optimal face value is set so that the total debt benefits are maximized:

$$P_p^* : \quad \frac{\partial DB(\nu_I, P)}{\partial P} = DB^P(\nu_I, P_p^*) = 0 \tag{55}$$

where P_p^* is the *ex post* optimal face value, and DB^P is the partial derivative function with respect to P.

The FOC for the optimal face value prior to issuance is more complicated. *Ex ante,* there are two decisions variables: the issuing threshold and the leverage (through face-value adjustments). The optimal threshold is a function of the face value prior to issuing debt. Having this in mind, the FOC for the optimal face value *ex ante*, P_a^*, is:

$$P_a^* : \quad \frac{\partial W(\nu_I(P), P)}{\partial P}|_{P_a^*} = 0 \iff \tag{56a}$$

$$0 = DB^P(\nu_I(P_a^*), P_a^*)(\frac{\nu}{\nu_I(P_a^*)})^{\beta_1} \tag{56b}$$

$$+\left[\nu_I^P(P_a^*)(\frac{\nu}{\nu_I(P_a^*)})^{\beta_1}\left(DB_I^{\nu}(\nu_I(P_a^*), P_a^*) - \beta_1 \frac{DB(\nu_I(P_a^*), P_a^*)}{\nu_I(P_a^*)}\right)\right] \tag{56c}$$

Now, this article claims $P_a^* = P_p^*$ by substituting P_p^* in Equation 56a. Term 56c is equal to zero because of the optimality condition of the threshold to issue debt (see Equation 4). Term 56b is also zero from Equation 55:

$$DB^P(\nu_I(P_a^*), P_a^*) = DB^P(\nu_I, P_p^*) = 0 \tag{57}$$

Therefore, P_p^* is a solution for Equation 56a. The equation has only one solution because DB^P is strictly decreasing and $\nu_I(P)$ is strictly increasing in P. Thus, the *ex ante* optimal face value (or leverage) is unique and equal to the *ex post* optimal face value, $P_p^* = P_a^*$. $\square$

I.7 The general conditions for co-existing optimal ZL policy and positive net benefits

I.7.1 Proposition 2 and the proof

Here, the appendix formally expresses the general properties of the debt benefit function, DB, in all the possible cases which debt issuance is feasible, but not optimal, and ZL policy exists. This generalizes the cases. If a trade-off model has assumptions matching with the proposition below, there is value in waiting to issue debt and the model can lead to ZL policy:

PROPOSITION 2. *If all the assumptions 1 to 4 hold, there exists an interval for the asset volatility values ($\sigma_I < \sigma < \sigma_{max}$) such that it is feasible to have debt but ZL policy and holding the real option to have debt is optimal, ceteris paribus.*

Proof. Feasibility means that the issuance creates positive net debt benefits and adds value to the firm. ZL policy means that immediate recapitalization with debt is not optimal and the firm is better off with keeping the real option. σ_{max} is the maximum volatility below which the debt issuance is feasible. σ_I is the volatility boundary above which waiting to issue debt is optimal. Due to the bijections discussed in Appendix F, the existence of the volatility boundary is equivalent to the existence of the unlevered asset threshold, ν_I. All the borderlines in the contour plots numerically calculate σ_I:

$$ZL = 1 \quad when \quad \sigma > \sigma_I(size, payoutrate, etc.), \quad and \quad ZL = 0, \quad otherwise \tag{58}$$

The assumptions and their explanations are:

Assumption 1. *The debt-benefit function (DB) is continuous and increasing in the firm's unlevered value:*

$$\frac{\partial DB(\nu)}{\partial \nu} \geq 0 \tag{59}$$

This assumption means that, everything else being equal, the net debt benefit increases if the firm's unlevered value increases. This is very intuitive since managers destroy the firm's value to gain on debt, if debt benefits decrease with an increasing unlevered value. Another explanation is that the chance to default declines when the unlevered value increases. Thus, tax savings rise, default costs fall, and the net debt benefits increase.

Assumption 2. *There exists a unique asset volatility level beyond which it is not feasible to issue debt and the net debt benefit is strictly decreasing at this point:*

$$\exists! \sigma_{max} : DB(\sigma_{max}, \nu_0) = 0, \quad \frac{\partial DB(\sigma, \nu_0)}{\partial \sigma}|_{\sigma=\sigma_{max}} < 0, \tag{60}$$

Based on Assumption 2, there is a limit for the net debt benefits of the firm. For volatility above this limit, σ_{max}, there is no positive gain in issuing debt. For example, with a very tiny fixed issuance cost, this assumption is valid and applies to Case I. Another example is Case II with fixed bankruptcy or issuance costs for which the assumption is valid.

Assumption 3. *The debt-benefit function and its derivative are continuous in the volatility.*

Assumption 4. *There exists at least one asset volatility level to satisfy Equation 4:*

$$\exists \sigma_x > 0 : J(\sigma_x) = 0, \qquad J(\sigma) = g(\sigma) - \beta_1 \frac{DB(\sigma, \nu_0)}{\nu_0}, \quad g(\sigma) = \frac{\partial DB(\sigma, \nu_0)}{\partial \nu_0} \tag{61}$$

The assumption here assures an existing solution to Equation 4. This assumption does not necessarily mean that there is a value to waiting, unless the earlier assumptions also hold. Now the assumptions above imply:

COROLLARY 1. *An asset volatility level, σ_I such that $\sigma_I = Max\{\sigma_x | J(\sigma_x) = 0\}$, exists and it is strictly smaller than σ_{max} ($\sigma_I < \sigma_{max}$).*

No asset volatility level larger than the maximum volatility, σ_{max}, satisfies Equation 4 ($\forall \sigma > \sigma_{max} : J(\sigma, \nu_0) > 0$). This result basically limits the value for the solution to Equation 4. From Assumption 2, the net debt benefit is strictly negative beyond the maximum feasible volatility ($\forall \sigma > \sigma_{max} : DB(\sigma, \nu_0) < 0$). In addition, Assumption 1 makes sure that the marginal benefits with respect to the unlevered value is also positive ($\forall \sigma > \sigma_{max} : g(\sigma, \nu_0) > 0$). $g(\sigma) - \beta_1 \frac{DB(\sigma, \nu_0)}{\nu_0}$ is strictly negative and $J(.)$ has no root beyond σ_{max}. Hence, σ_I exists and cannot be larger or equal to maximum volatility.

COROLLARY 2. *For asset volatility between σ_{max} and σ_I ($\sigma_I < \sigma < \sigma_{max}$): $J(\sigma) > 0$.*

Since σ_I is a solution to $J(\sigma_x) = 0$, there are two possibilities :

$$\begin{cases} either : J(\sigma_I < \sigma < \sigma_{max}) > 0, \quad and \quad J(\sigma_I > \sigma) < 0 \\ or : \quad J(\sigma_I < \sigma < \sigma_{max}) < 0, \quad and \quad J(\sigma_I > \sigma) > 0 \end{cases} \tag{62}$$

Otherwise, there exists another root for $J(\sigma) = 0$ which is larger than σ_I, and it contradicts the definition of σ_I in Corollary 1. Only, $J(\sigma_I < \sigma < \sigma_{max}) > 0$ holds. $J(\sigma_{max})$ is continuous and positive (from Corollary 1) because it is a sum of two continuous functions. Therefore, there exists a left reduced neighborhood around σ_{max} where the function J is still positive. This means all the volatilities in the range are also positive:

$$\begin{aligned} &J(\sigma_{max}) > 0 \Rightarrow J(\sigma \to \sigma_{max}^{-}) > 0 \Rightarrow \exists \sigma_a : J(\sigma_a < \sigma < \sigma_{max}) > 0 \\ &\iff J(\sigma_I < \sigma < \sigma_{max}) > 0 \end{aligned} \tag{63}$$

Since $J(.)$ can only take positive values in this interval, the marginal debt value is higher than the average debt benefits, while the net debt benefit for these volatility levels is positive:

$$J(\sigma_I < \sigma < \sigma_{max}) > 0 : \frac{\partial DB(\sigma, \nu_0)}{\partial \nu_0} > \beta_1 \frac{DB(\sigma, \nu_0)}{\nu_0}, \quad and \quad DB(\sigma, \nu_0) > 0 \tag{64}$$

Therefore, there are volatility levels at which it is feasible to issue debt, $DB(.) > 0$ but postponing the issuance is optimal ($\partial DB/\partial\nu > \beta_1 DB/\nu$). $\square$

The cases are examples of the models with similar assumptions. The empirical estimation also assumes that the debt structure of ZL firms in data matches with the assumptions in Proposition 2. Moreover, the proof shows that the volatility boundary exists under certain conditions. Due to the bijection between volatility boundary and the asset threshold shown in Appendix F, it also proves that the asset threshold exists.

I.8 Case III: The bankruptcy cost split

This appendix extends Case II and only presents the valuation formulas. In the earlier cases, debt holders pay the default cost *ex post*. For many ventures and young firms, leaving entrepreneurs also pay some costs when they leave the firm due to bankruptcy. Example of the entrepreneur's loss is the lost legacy for the entrepreneur's descendants at default. This inefficiency splits the bankruptcy costs between equity holders (including entrepreneurs) and debt holders. In order to capture these dynamics, I change the model in Case I and check its implications. Mathematically, the cost is split between debt holders and equity holders proportionally (bp represents the proportion of the cost paid by the shareholders depending on the bargaining power of debt holders). Limited liability also holds.

The debt gain and the real option are:

$$
\begin{aligned}
&DB_{bp}(\nu) = TS_{bp}(\nu) - BC_{bp}(\nu) \\
&W_{bp}(\nu) = DB_{bp}(\nu_I^{bp})(\frac{\nu}{\nu_I^{bp}})^{\beta_1} \qquad \nu \leq \nu_I^{bp}
\end{aligned}
\tag{65}
$$

where ν_I^{bp} is the threshold to issue debt, DB_{bp} is the net debt benefit, and W_{bp} is the real option value. The optimal threshold maximizes the option value and satisfies a similar equation as Equation 4:

$$\nu_I^{bp} \quad s.t. \quad \frac{\partial DB(\nu_I^{bp})}{\partial \nu_I^{bp}} = \beta_1 \frac{DB(\nu_I^{bp})}{\nu_I^{bp}} \tag{66}$$

Each claim's value is:

$$D_{bp}(\nu) = \frac{C}{r} + ([\nu_B^{bp} - (1-bp)[\alpha\nu_B^{bp} + K]]^+ - \frac{C}{r})(\frac{\nu}{\nu_B^{bp}})^{-\beta_2} \tag{67}$$

$$TS_{bp}(\nu) = \frac{\tau C}{r}(1 - (\frac{\nu}{\nu_B^{bp}})^{-\beta_2}) \tag{68}$$

$$BC_{bp}(\nu) = (\alpha\nu_B^{bp} + K)(\frac{\nu}{\nu_B^{bp}})^{-\beta_2}) \tag{69}$$

$$E_{bp}(\nu) = \nu + TS_{bp}(\nu) - BC_{bp}(\nu) - D_{bp}(\nu) \tag{70}$$

$$\frac{\partial E_{bp}}{\partial \nu_B^{bp}} = 0 \Rightarrow \nu_B^{bp} = \frac{(C/r) - \tau(C/r) - bpK}{(1 + bp\alpha)}(\frac{\beta_2}{1+\beta_2}) \tag{71}$$

Equation 71 shows the barrier in the new model. The optimal default barrier has a more general form.[33] Shutting down the cost sharing generates the same results as in Case II. As the portion paid by the shareholders gets larger, they lower the default barrier to avoid the cost paid at default. The same is true when their share is fixed and the PBC rate or the fixed default cost increases. This implies that any undesirable cost makes the shareholders postpone exercising the default option.

[33]In order to drop the positive portion sign, $bpK \leq (1-\alpha)\nu_B^{bp}$ holds .